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A DISEQUILIBRIUM THEORY OF LONG-RUN PROFITS:
SCHUMPETERIAN DYNAMICS

by

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December 1998

<ABSTRACT>

In the traditional economic theory, whether classical or neoclassical, the long-run state is an equilibrium state where all the profits in excess of normal rate are completely wiped out. If there is a theory of long-run profits, it is a theory about the determination of the normal profit rate. This paper challenges this long-held tradition in economics. It develops a simple evolutionary model of industry dynamics and demonstrates that what the economy approaches in the long-run is not a classical or neoclassical equilibrium of uniform technology but (at best) a statistical equilibrium of technological disequilibria which reproduces a dispersion of efficiencies in a statistically balanced form. As Schumpeter once remarked, "surplus values may be impossible in perfect equilibrium, but can be ever present because that equilibrium is never allowed to establish itself." The paper also shows that this evolutionary model can calibrate all the macroscopic characteristics of neoclassical growth model without the neoclassical assumption of optimizing decision-makings.

<Keywords> evolutionary economics, Schumpeter, disequilibrium, theory of profits, industry dynamics, technological change, growth accounting.

<JEL classification numbers> L10, O30, D50, P10.

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1. Introduction.

The title of this paper may sound a contradiction in terms. In the traditional economic theory, by which I include both classical and neoclassical economics, the long-run state of an economy is an equilibrium state and the long-run profits (if they ever exist) are equilibrium phenomena. *Fig.1* illustrates this by drawing two supply curves that can be found in any textbook of economics. In the upper panel is an upward-sloping supply curve which aggregates diverse cost conditions of the existing firms in an industry. Its intersection with a downward-sloping demand curve determines an equilibrium price, which in turn determines the amount of profits (represented by the shaded triangle) accruing to the industry as a whole. As long as the supply curve is upward-sloping, an industry is able to generate positive profits.

<Insert *Fig.1* around here.>

In traditional theory, however, this is merely a description of the 'short-run' state of an industry. Whenever there are positive profits, existing firms are encouraged to expand their productive capacities and potential firms are induced to enter the industry, both making the supply curve flatter and flatter. This process will continue until the industry supply curve becomes totally horizontal, thereby wiping out any opportunity for positive profits. The lower panel of *Fig.1* describes this 'long-run' state of the industry.

This implies that if there are any profits in the long-run, it must be the 'normal' profits which have already been incorporated into cost calculations. In fact, it is how to explain the fundamental determinants of these normal profits which divides the traditional economic theory into classical and neoclassical approaches. Classical economics (as well as Marxian economics) has highlighted an inverse relationship between the normal profit rate and the real wage rate, and reduced the problem of determining the former to that of determining the latter and ultimately to that of distributional conflicts between classes. Neoclassical economics has identified the normal profit rate with the interest rate plus a risk premium and reduced the problem of its determination to that of characterizing

equilibrium conditions for intertemporal resource allocation under uncertainty. But no matter how opposed their views might appear over the ultimate determinants of normal profits, they share the same 'equilibrium' perspective on long-run profits -- any profits in excess of the normal rate are 'disequilibrium' phenomena which are bound to disappear in the long-run.

   It is Joseph Schumpeter who gave us a powerful alternative to this deep-rooted 'equilibrium' tradition in the theory of long-run profits. According to Schumpeter, it is through an "innovation" or "doing things differently" that positive profits emerge in the capitalist economy. "The introduction of new commodities..., the technological change in the production of commodities already in use, the opening-up of new markets or of new sources of supply, Taylorization of work, improved handling of material, the setting-up of new business organizations"[1] etc. allow the innovators to charge prices much higher than costs of production. Profits are thus the premium put upon innovation. Of course, the innovator's cost advantage does not last long. Once an innovation is successfully introduced into the economy, "it becomes much easier for other people to do the same thing."[2] A subsequent wave of imitations soon renders the original innovation obsolete and gradually wears out the innovator's profit rate. In the long-run, there is therefore an inevitable tendency towards classical or neoclassical equilibrium which does not allow any positive profits in excess of the normal rate. And yet Schumpeter argued that positive profits will never disappear from the economy because capitalism is "not only never but never can be stationary." It is an "evolutionary process" that "incessantly revolutionizes the economic structure *from within*, incessantly destroying an old one, incessantly creating a new one."[3] Indeed, it is to destroy the tendency towards classical or neoclassical equilibrium and to create a new industrial disequilibrium that is the function the capitalist economy has assigned to those who carry out innovations. "Surplus values [i.e., profits in excess of normal rate] may be impossible in perfect equilibrium, but can be ever present because that equilibrium is never allowed to establish itself.

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[1] Schumpeter [1939], p.84.
[2] Schumpeter [1939], p.100.
[3] Schumpeter [1950], p. 83.

They may always tend to vanish and yet be always there because they are incessantly recreated."[4]

It is the first objective of this paper to formalize this grand vision of Joseph Schumpeter from the perspective of evolutionary economics.[5] It makes use of a simple evolutionary model of Iwai [1984a, b] to demonstrate the Schumpeterian thesis that profits in excess of normal rate will never disappear from the economy no matter how long it is run. Indeed, it will be shown that what the economy will approach over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but (at best) a statistical equilibrium of technological disequilibria which reproduces a relative dispersion of efficiencies among firms in a statistically balanced form. Although positive profits are impossible in perfect equilibrium, they can be ever present because that equilibrium is never allowed to establish itself.

This paper is organized as follows. After having set up the static structure of an industry in section 2, the following three sections will develop an evolutionary model of industrial dynamics and examine how the firms' capacity growth, technological imitations and technological innovations respectively move the industry's state of technology over time. It will be argued that while both the differential growth rates among different efficiency firms and the diffusion of better technologies through imitations push the state of technology towards uniformity, the punctuated appearance of technological innovations disrupts this equilibriating tendency. Section 6 will then turn to the long-run description of the industry's state of technology. It will indeed be shown that over a long passage of time these conflicting microscopic forces will balance each other in a statistical sense and give rise to a long-run distribution of relative efficiencies across firms. This long-run distribution will in turn allow us to deduce an *upward-sloping* long-run supply curve in section 7. The industry is thus capable of generating positive profits even in the long-run! Hence, the title of this paper –– 'a disequilibrium theory of long-run profits'.

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[4] Schumpeter [1950], p. 28.
[5] See, for instance, Nelson and Winter [1982], Dossi, Freeman, Nelson, Silverberg and Soete [1988], Metcalfe and Saviotti [1991], and Anderson [1994] for the comprehensive expositions of the "evolutionary perspective" in economics.

Section 8 will then examine the factors determining the long-run profit rate of the industry.

The present paper will adopt the 'satisficing' principle for the description of firms' behaviors – firms do not optimize a well-defined objective function but simply follow organizational routines in deciding their growth, imitation and innovation policies.[6] Indeed, the purpose of the penultimate section 9 is to show that our evolutionary model is able to "calibrate" all the macroscopic characteristics of neoclassical growth model without having recourse to the neoclassical assumption of fully optimizing economic agents. If we look only at the aggregative performance of our evolutionary economy, it is as if aggregate labor and aggregate capital together produce aggregate output in accordance with a well-defined aggregate production function with Harrod-neutral technological progress. Yet, this macroscopic picture is a mere statistical illusion. If we zoomed into the microscopic level of the economy, what we would find is the complex and dynamic interactions among many a firm's capital growth, technological imitations and technological innovations. It is simply impossible to group these microscopic forces into a movement along an aggregate production function and a shift of that function itself. The neoclassical growth accounting may have no empirical content at all.

Section 10 concludes the paper.

2. Construction of the industry supply curve.

The starting point of our evolutionary model is an observation that knowledge is not a public good freely available among firms and that technologies with a wide range of efficiency coexist even in the same industry. And one of the end points of our evolutionary model is to demonstrate that technologies with a wide range of efficiency will indeed coexist even in the long-run.

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[6] The term "satisficing" was first coined by Simon [1957] to designate the behavior of a decision maker who does not care to optimize but simply wants to obtain a satisfactory utility or return. The notion of "organizational routines" owes to Nelson and Winter [1982]. Organizations "know" how to do things. In Iwai [1999] I have provided a legal-economic-sociological framework for understanding the nature and sources of such organizational capabilities.

Consider an industry which consists of many firms producing the same product.[7] Let us denote by n the total number of technologies coexisting in the industry and assume that each of these technologies is of Leontief-type fixed-proportion technology with labor service as the sole variable input and capital stock as the sole fixed input. If we further assume that only labor productivity varies across technologies, we can express the ith technology as:

(1) $\quad q = Min[\dfrac{l}{c_i}, \dfrac{k}{b}]$,

where q, l and k represent final output, labor input and capital stock, and c_i and b are labor and capital coefficients. Let us choose money wage as the numeraire. Then, the labor coefficient c_i determines the unit cost of each technology up to a productive capacity k/b. I will slightly abuse the term and call c_i the 'unit cost' of technology i. It is then possible to rearrange the indices of technology and let c_n stand for the lowest and c_1 the highest unit cost of the industry without an loss of generality, or:

(2) $\quad c_n < c_{n-1} < ... < c_i < ... < c_1$.

I now have to introduce several notations in order to construct the supply curve of the industry in question. Let $k_t(c_i)$ represent the sum of the capital stocks of all the firms whose unit cost is c_i at time t, and let $K_t(c_i) \equiv k_t(c_n)+...+k_t(c_i)$ represent the cumulative sum of all the capital stocks of the firms whose unit costs are c_i or lower at time t. The industry's total capital stock at time t can then be represented by $K_t(c_1)$, but will be denoted simply as K_t in the following discussion. Next, let $s_t(c_i)$ and $S_t(c_i)$ represent the 'capital share' and the 'cumulative capital share' of a unit cost c_i at time t. Of course, we have $s_t(c_i) \equiv k_t(c_i)/K_t$ and $S_t(c_i) \equiv K_t(c_i)/K_t$. As a convention, we set $S_t(c) = S_t(c_i)$ for $c_i \leq c < c_{i-1}$. *Fig.2* exhibits a typical distribution of cumulative capital shares in the industry. It illustrates the 'state of technology' at a point in time by showing us how technologies with diverse unit costs are distributed among capital stocks of an the industry.

<center><Insert Fig.2 around here.></center>

The state of technology thus introduced, however, represents merely the production 'possibility' of an industry. How this possibility is actualized depends upon the price each firm is able to obtain in exchange for its product. Let us assume that the industry in question is a competitive

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[7] Or we can think of this as a one-commodity economy with many competing firms.

industry in which a large number of firms are producing the same homogeneous product and charge the same price for it.[8] Let us denote by P_t the product price (measured in terms of money wage) at time t. Then, under the assumptions of homogeneous product and fixed proportion technology, firms with unit costs strictly smaller than P_t decide to produce up to their productive capacity k/b, and firms whose unit costs are strictly higher than P_t decide to quit all production. Firms with the unit cost equal to P_t are indifferent to their production level, as long as it does not exceed their productive capacity. (We ignore here the cost of shutting-down of a factory as well as the cost of setting-up of a new production line.)

It follows that when $c_{i-1} > P_t > c_i$ the total supply of the industry product becomes equal to $K_t(c_i)/b$ and that when $P_t = c_i$ it takes any value from $K_t(c_{i+1})/b$ to $K_t(c_i)/b$. Hence, if we denote by $Y_t(P)$ the industry's 'short-run supply curve' (or short-run supply correspondence, to be precise) at time t, it can be written as

(3) $Y_t(P) = K_t(c_i)/b$ if $c_i < P < c_{i-1}$

          $\varepsilon \ [K_t(c_{i+1})/b, \ K_t(c_i)/b]$ if $P = c_i$.

Dividing this by the total productive capacity K_t/b, we can also express it as:

(4) $y_t(P) \equiv Y_t(P)b/K_t \ = S_t(c_i)$ if $c_i < P < c_{i-1}$

          $\varepsilon \ [S_t(c_{i+1}), \ S_t(c_i)]$ if $P = c_i$.

(4) is nothing but the 'relative' form of industry supply curve at time t, which has neutralized the scale effect of changes in the total capital stock of the industry. Since the forces governing the motion of $S_t(c)$ are in general of different nature from those governing the motion of K_t, I will be concerned mostly with this relative form of industry supply curve in what follows.[9]

<Insert Fig.3 around here.>

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[8] Our evolutionary model can also accommodate a wide variety of industry structures. See Appendix A of Iwai [1984b] for the way to deal with the case of monopolistically competitive industry.

[9] It is easy to show from (8) below that: $\dot{K_t}/K_t \ = \ \gamma(log \ P_t - \sum_i(log \ c_i)s_t(\ c_i)) - \gamma_0$, so that the growth rate of the industry's total capital stock is linearly dependent on the proportional gap between the price-wage ratio P_t and the industry-wide average unit cost. If $\dot{K_t}/K_t$ is pre-determined (probably by the growth rate of the demand for this industry's products), this equation can be used to determine P_t. If, on the other hand, P_t is pre-determined (probably by the labor market conditions in the economy as a whole), this equation can be used to determine $\dot{K_t}/K_t$. In either case, the forces governing the motion of K_t are in general of the different nature from those governing the evolution of $\{S_t(c)\}$.

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Fig.3 depicts the relative form of industry supply curve, $y_t(P) \equiv Y_t(P)b/K_t$, in a Marshallian diagram with prices and costs (both in terms of money wage) measured along vertical axis and quantities per unit of total productive capacity measured along horizontal axis. Indeed, it merely turns *Fig.2* around 45 degree line. It is an *upward-sloping* curve as long as different unit costs coexist within the same industry.

3. Darwinian dynamics of the state of technology.

Any freshman knows that the industry supply curve is a horizontal sum of all the individual supply curves existing in the industry. But the problem we now have to tackle with is to ascertain how the dynamic competition among firms will mold the evolutionary pattern of the supply curve and govern the fate of the industry. This is not the problem for freshman. Since there is a one-to-one correspondence between the relative form of industry supply curve and the cumulative distribution of capital shares, the analysis of the dynamic evolution of the former can be reduced to that of the latter.

Now, the state of technology in our Schumpeterian industry is moved by complex interactions among the dynamic forces working at the microscopic level of individual firms -- successes and failures of technological innovations and imitations and the resulting differential growth rates among competing firms. Let us examine the effect of differential growth rates first.

"Without development there is no profit, without profit no development," so said our Schumpeter.[10] The following hypothesis relates the growth rate of capital stock to the rate of profit:

Hypothesis (CG): The capital growth rate of a firm with unit cost c_i is
  linearly increasing in its current rate of profit $r_t(c_i)$, or it is equal to:

(5) $\gamma r_t(c_i) - \gamma_0$;

  where $\gamma > 0$ and $\gamma_0 > 0$ are given constants. □

This hypothesis needs little explanation. It merely says that a higher profit rate on the existing capital stocks stimulates capital accumulation, either by influencing the expected profitability of new investment projects or by directly providing an internal fund for the projects. The parameter γ (or, more precisely, γ/b) represents the sensitivity of the firm's growth rate to the current profit rate, and the parameter γ_0 represents the rate of capital

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[10] Schumpeter [1961], p. 154.

depreciation of the break-even firm. As I have already indicated in section 1, the present paper follows the strict evolutionary perspective in supposing that firms do not optimize but only 'satisfice' in the sense that they simply follow organizational routines in deciding their growth, imitation and innovation policies. Indeed, one of the purposes of this paper is to see how far we can go in our description of the economy's dynamic performance without relying on the assumptions of individual optimality. I will therefore assume that the values of γ and γ_0 are both exogenously given.[11]

We have already assumed that every firm in the industry produces the same homogeneous product and faces the same price P_t. If we further assume that the price of capital equipment is proportional to P_t, we can calculate the profit rate $r(c_i) \equiv (P_t y_t - c_i l_t)/P_t k_t$ as $b(P_t - c_i)/P_t$, which we will approximate as $b(\log P_t - \log c_i)$ for analytical convenience. Then, by simply differentiating the cumulative capacity share $S_t(c_i)$ with respect to time, *Hypothesis (CG)* allows us to deduce the following set of differential equations for the dynamics of cumulative capital shares[12]:

(6) $\dot{S}_t(c_i) = \gamma \delta_t(c_i) S_t(c_i)(1 - S_t(c_i))$ $\qquad (i = n, n-1, ..., 1).$

In the above equations, $\delta_t(c_i)$ represents the difference between the logarithmic average of a set of unit costs higher than c_i and the logarithmic average of a set of unit costs not higher than c_i, or:

(7) $\delta_t(c_i) \equiv \sum_{j=1}^{i-1} \dfrac{(\log c_j) s_t(c_j)}{1 - S_t(c_i)} - \sum_{j=i}^{n} \dfrac{(\log c_j) s_t(c_j)}{S_t(c_i)} > 0.$

Its value in general depends on t and the whole distribution of c_i. I will, however, proceed the following analysis as if it were an exogenously given constant δ, uniform both across technologies and over time. This will

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[11] It is, however, not so difficult to deduce an investment function of this form by explicitly setting up an intertemporal optimization problem with adjustment costs, as in Uzawa [1969].

[12] The actual derivation is as follows. $\dot{S}_t(c_i) \equiv \sum_{j=i}^{n} \dot{s}_t(c_j)$

$= \sum_{j=i}^{n} (\dot{k}_t(c_j)/k_t(c_j) - \dot{K}_t/K_t) s_t(c_j)$

$= \sum_{j=i}^{n} ((\gamma(\log p_t - \log c_j) - \gamma_0) - \sum_{h=1}^{n} (\gamma(\log p_t - \log c_h) - \gamma_0) s_t(c_h)) s_t(c_j)$ by (5)

$= \sum_{j=i}^{n} \gamma (\sum_{h=1}^{n} (\log c_h) s_t(c_h) - \log c_j) s_t(c_j)$

$= \gamma (\sum_{h=1}^{i-1} (\log c_h) s_t(c_h) S_t(c_i) + \sum_{h=i}^{n} (\log c_h) s_t(c_h) S_t(c_i) - \sum_{j=i}^{n} \log c_j s_t(c_j))$

$= \gamma \delta_t(c_i) S_t(c_i)(1 - S_t(c_i))$.

simplify the exposition of our evolutionary model immensely without losing any of its qualitative nature.[13] Then, we can rewrite *(6)* as:

(8) $\dot{S}_t(c_i) \equiv \gamma \delta S_t(c_i)(1 - S_t(c_i))$ $(i = n, n-1, ..., 1)$.

Each of the above equations is a well-known 'logistic differential equation' with a logistic parameter μ, and can be solved explicitly to yield:

$$(9)\ S_t(c_i) = \frac{1}{1 + (1/S_T(c_i) - 1)e^{-\gamma\delta(t-T)}}\qquad (i = n, n-1, ..., 1),$$

where *e* stands for the exponential and $T\ (\leq t)$ a given initial time.[14]

<center><Insert *Fig.4* around here.></center>

Differential growth rates among firms with different cost conditions never leave the industry's state of technology static. As the firms with relative cost advantage grow faster than the firms with relative cost disadvantage, the distribution of capital shares gradually shifts in favor of the lower unit costs, thereby reducing the average unit cost of the industry as a whole. This process then eliminates the relative cost advantage of the existing technologies one by one until the capital share of the least unit cost completely overwhelm those of the higher ones. Only the fittest will survive in the long-run through their higher growth rates, and this of course is an economic analogue of 'Darwinian' natural selection mechanism. The set of logistic equations *(9)* describes this 'economic selection' mechanism in the simplest possible mathematical form, and its evolutionary dynamics is illustrated by *Fig. 4*. In particular, the equation for *i = n* shows that the cumulative capital share of the lowest unit cost $S_t(c_n)$ moves along an *S*-shaped growth path. It grows almost exponentially when it occupies a negligible portion of the industry, gradually loses its growth momentum as its expansion narrows its own relative cost advantage, but never stops growing until it swallows the whole industry.

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[13] This is the simplification I also adopted in Iwai [1984b]. However, in a recent article Franke [1998] indicated that the value of $\delta_i(c)$ may actually vary considerably as the parameter values of γ, ν and λ as well as the value of *c* vary. A caution is thus needed to use this approximation for purposes other than heuristic device.

[14] A logistic differential equation: $x' = ax(1-x)$ can be solved as follows. Rewrite it as: $x'/x - (1-x)'/(1-x)$ and integrate it with respect to *t*, we obtain: $\log(x) - \log(1-x) = \log(x_0) - \log(1-x_0) + at$, or $x/(1-x) = e^{at}x_0/(1-x_0)$. This can be rewritten as: $x = 1/(1+(1/x_0-1)e^{-at})$, which is nothing but a logistic equation given by *(9)*.

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4. Lamarkian dynamics of the state of technology

Next, let us introduce the process of technological imitation and see how it molds the dynamics of the state of technology. In this paper I will suppose that technology is not embodied in capital stocks and hypothesize the process of imitations as follows[15]:

Hypothesis (IM'): The probability that a firm with unit cost c_i succeeds in imitating a technology with unit cost c_j is equal to:

(10) $\mu s_t(c_j)dt$ if $c_j < c_i$ and 0 if $c_j \geq c_i$,

for a small time interval dt; where $\mu \, (> 0)$ is assumed to be a constant uniform across firms. ◊

One of the characteristic features of technology is its non-excludability. It may be legally possible to assign property rights to the owners of technology. But, as Arrow has remarked in his classic paper [1962], "no amount of legal protection can make a thoroughly appropriable commodity of something so intangible as information," because "the very use of the information in any productive way is bound to reveal it, at least in part."[16] The above hypothesis mathematically captures such spill-over effects of technology in the simplest possible manner. It says that it is much easier for a firm to imitate a technology with high visibility (i. e., a large capital share), than to imitate a technology with low visibility (i.e., a small capital share). Needless to say, the firm never imitates the technology whose unit cost is not smaller than its current one. The imitation coefficient μ in the above hypothesis represents the effectiveness of each firm's imitative activity. There is a huge body of literature, both theoretical and empirical, which identifies factors which influence the effectiveness of firms' imitation activities.[17] The main concern of the present paper is, however, to work out

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[15] The reason I have designated this *Hypothesis* by *(IM')* is to differentiate it from a slightly different hypothesis adopted in Iwai [1984a]. Its *Hypothesis (IM)* assumes that the probability of imitating a better technology is proportional to the frequency (rather than their capital share) of the firms using it. On the other hand, Iwai [1998] has adopted yet another hypothesis which assumes that firms imitate only the best practice technology and the probability of its success is proportional to the frequency of the firms using it.

[16] p. 615.

[17] See, for instance, Mansfield, Schwartz and Wagner [1981], Gorts and Klepper [1982] and Metcalfe [1988].

the dynamic mechanism through which a given imitation policy of firms structures the evolutionary pattern of the industry's state of technology. In what follows I will simply assume that μ is an exogenously given parameter, uniform across firms and constant over time.[18]

In order to place the effect of technological diffusion in full relief, let us ignore the effect of economic selection for the time being. Then, the hypothesis *(IM')* allows us to deduce the following set of logistic differential equations as a description of the evolution of the state of technology under the sole pressure of technological imitations[19]:

$$(11) \quad \dot{S}_t(c_i) = \mu S_t(c_i)(1 - S_t(c_i)) \qquad (i = n, n\text{-}1, ..., 1) .$$

We have again encountered logistic differential equations, which can then be solved to yield the second set of logistic equations in this paper!

$$(12) \quad S_t(c_i) = \frac{1}{1 + (1/S_T(c_i)\text{-}1)e^{-\mu(t-T)}} \qquad (i = n, n\text{-}1, ..., 1),$$

where $T (\leq t)$ a given initial time.

Since the second set of logistic equations *(12)* is mathematically equivalent to the first set of logistic equations *(9)*, *Fig. 4* in the preceding section can again serve to illustrate the dynamic evolution of the cumulative capacity shares under the sole pressure of technological diffusion. And yet, the logic behind these second logistic equations is entirely different from that of the first. "If one or a few have advanced with success many of the difficulties

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[18] It is, however, possible to incorporate a trade-off between the resources devoted to capital growth and the resources devoted to imitative activities into our model. For instance, the growth parameters γ and/or $-\gamma_0$ in (5) can be made a decreasing function of the imitation coefficient μ.

[19] The actual derivation is as follows. The value of $S_t(c_i)$ increases whenever one of the firms with unit costs higher than c_i succeeds in imitating one of the technologies with unit costs c_i or lower. Indeed, because of the assumption of the disembodied nature of technology, it increases by the magnitude equal to the imitator's capacity share. Note that $S_t(c_i)$ is not affected by the imitation of any of the firms with unit costs c_i or less, for it only effects an infra-marginal transfer of capacity share. Let $M_t(c_i)$ denote the number of firms with unit costs c_i or lower. Since the average capacity share of the firms with unit costs higher than c_i is $(1-S_t(c_i))/(M-M_t(c_i))$ and the probability of a successful imitation for each of those $M-M_t(c_i)$ firms is $\mu S_t(c_i)dt$ during a small time interval dt, we can calculate the expected increase in $S_t(c_i)$ during dt as $((1-S_t(c_i))/(M-M(c_i))(\mu S_t(c_i)dt)(M-M_t(c_i)) = (\mu S_t(c_i)dt)(1-S_t(c_i))$. If the number of firms is sufficiently large, the law of large numbers allows us to use this expression as a good approximation of the actual rate of change in $S_t(c_i)$. Dividing this by dt and letting $dt \to 0$, we obtain *(11)*.

disappear," so wrote Schumpeter, "others can then follow these pioneers, as they will clearly do under the stimulus of the success now attainable. Their success again makes it easier, through the increasingly complete removal of the obstacles..., for more people follow suit, until finally the innovation becomes familiar and the acceptance of it a matter of free choice."[20] The logistic equations *(12)* describe this swarm-like appearance of technological imitations in the simplest possible form. In particular, the equation for $i = n$ shows that the cumulative capital share of the lowest unit cost moves along a *S*-shaped growth path, initially growing at an exponential rate but gradually decelerating its growth rate to approach unity asymptotically. In the long-run, therefore, the lowest cost technology will dominate the whole industry, simply because it will eventually be diffused to all the firms in it. This technological diffusion process is nothing but an economic analogue of the 'Lamarkian' model of biological evolution – the achievement of one individual are passed directly to the other individuals.

Let us then bring back the Darwinian process of economic selection into our industry and add *(6)* to *(11)*. The result is the third set of logistic differential equations in the present paper:

(13) $\dot{S}_t(c_i) = (\gamma\delta+\mu)S_t(c_i)(1-S_t(c_i))$ $\qquad\qquad$ *(i = n, n-1, ..., 1)*,

which can again be solved explicitly as:

(14) $S_t(c_i) = \dfrac{1}{1+(1/S_T(c_i)-1)e^{-(\gamma\delta+\mu)(t-T)}}$ $\qquad$ *(i = n, n-1, ..., 1)*,

for $t \geq T$. (We refrain from drawing a diagram for the third set of logistic equations *(14)* which is qualitatively the same as *Fig. 4*.)

We have thus shown how the mechanism of economic selection and the process of technological diffusion jointly contribute to the logistic growth process of cumulative capital shares -- the former by amassing the industry's capacities in the hands of the lowest cost firms and the latter by diffusing the advantage of the lowest cost technology among imitating firms. While the former is Darwinian, the latter is Lamarkian. But, no matter how opposed the underlying logic might be, their effects upon the industry's state of technology are the same --- the lowest cost technology will eventually dominate the whole capital stocks of the industry.

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[20] Schumpeter [1961], p. 228.

5. Punctuated dynamics of the state of technology.

Does this mean that the industry's long-run state is no more than the paradigm of classical and neoclassical economics in which every market participant is supposed to have a complete access to the most efficient technology of the economy?

The answer is, however, "No". And the key to this negative answer lies, of course, in the phenomenon of innovation -- the carrying out of what Schumpeter called a "new combination." Indeed, the functional role of innovative firms is precisely to destroy this tendency towards static equilibrium and to create a new industrial disequilibrium.

Suppose that at some point in time one of the firms succeeds in introducing a new technology with unit cost c_{n+1} smaller than c_n. Let us denote this time by $T(c_{n+1})$ and call it the 'innovation time' for c_{n+1}. Then, a new cumulative capital share $S_t(c_{n+1})$ emerges out of nothing at $T(c_{n+1})$. Because of the disembodied nature of technology, $S_{T(cn+1)}(c_{n+1})$ is identical with the capital share of the innovator of c_{n+1}. Moreover, if the innovator's unit cost was, say, c_i before innovation, all the cumulative capital shares from $S_t(c_{i+1})$ to $S_t(c_n)$ also experience a jump of the same magnitude at time $T(c_{n+1})$. In no time the innovator starts to expand its capital stocks rapidly, which then induces all the other firms to seek opportunities to imitate its technology. Through such selection mechanism and diffusion process, the newly created cumulative capital share begins to follow a *S*-shaped growth curve described by *(14)*.

<Insert *Fig. 5* around here.>

Innovation is not a single-shot phenomenon, however. No sooner than an innovation occurs, a new round of competition for a better technology begins. And no sooner than a new winner of this game is named, another round of technological competition is set out. The process repeats itself forever, and technologies with ever lower unit costs, $c_{n+2} > c_{n+3} > ... > c_N > ...$ will be introduced into an industry one by one at their respective innovation times $T(c_{n+2})$, $T(c_{n+3}),..., T(c_N)$, ...

*Fig.*5 shows how the industry's state of technology evolves over time, now as an outcome of the interplay among three dynamical forces working in the industry -- economic selection mechanism, technological diffusion through imitations and creative destruction of innovations. In fact, while the former

two work as equilibriating forces which tend the state of technology towards uniformity, the third works as a disequilibriating force which destroys this leveling tendency. A new question then arises: is it possible to derive any law-like properties out of this seemingly erratic movement of the industry state of technology?

In order to give an answer to this question, it is necessary to introduce two more hypotheses -- one pertaining to invention and the other to innovation. The conceptual distinction between invention and innovation was very much emphasized by Schumpeter. Invention is a discovery of new technological possibility which is potentially applicable to the production processes of the economy. But, "as long as they are not carried into practice," so says Schumpeter, "inventions are economically irrelevant," and "to carry any improvement into effect is a task entirely different from the inventing of it."[21]

Denote then by $C(t)$ the unit cost of potentially the best possible technology at time t and call it 'the potential unit cost'. The following is our hypothesis about the process of inventions[22]:

Hypothesis (PC): The potential unit cost is declining at a positive constant rate λ over time.

(15) $C(t) = e^{-\lambda t}$,

where the scale of $C(0)$ is chosen to be unity. □

The declining rate of potential unit cost λ reflects the speed at which the stock of technological knowledge is being accumulated by academic institutions, private firms, government agencies and amateur inventors throughout the entire economy. In the present paper which follows an evolutionary perspective, however, it is assumed to be given exogenously to the industry.

We are then able to characterize the notion of 'innovation' formally as an event in which the potential unit cost is put into actual use by one of the firms in the industry. This is tantamount to saying that when an innovation takes place at time t, it brings in a technology of unit cost $C(t)$ for the first time into an industry. This also implies that if a technology with unit cost c

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[21] Schumpeter [1961], p. 88.
[22] Iwai [2000], however, presents an evolutionary model which does not separate innovators from inventors and assume that each innovation raises the productivity of the industry's best technology by a fixed proportion.

is presently in use, it must have been introduced at time $t = T(c)$ where $T(c)$ is the inverse function of $C(t)$ defined by:

(16) $T(C(t)) \equiv t$ or $C(T(c)) \equiv c$.

The function $T(c)$ thus defined is nothing but the 'innovation time' for unit cost c we have already defined at the beginning of this section. Under the specification of the dynamics of potential unit cost in *(15)*, we have $T(c) = -(logc)/\lambda$.

Next, let us introduce the hypothesis about the process of innovations:

Hypothesis (IN-a): The probability that a firm succeeds in an innovation is equal to:

(17) vdt ,

during any small time interval *dt*, where v is a small positive constant.□

The parameter v represents the effectiveness of each firm's innovative activity in the industry. Its value should in general reflect a particular innovation policy the firm has come to adopt in its long-run pursuit for technological superiority, and there is a huge body of literature identifying the factors which influence the firm's innovation policy.[23] Our main concern in the present article, however, is rather to examine how a given innovation policy will mold the evolutionary pattern of the industry's state of technology in the long-run. In what follows I will simply assume that v is an exogenously given parameter, uniform across firms and constant over time.[24]

Implicit in the above hypothesis is the supposition that an innovation can be introduced at any time and by any firm, irrespective of at what time and by which firm the last innovation was introduced.[25] Indeed, if we let M denote the total number of firms in the industry, the probability that there is an innovation during a small time interval *dt* is equal to $(vdt)M = vMdt$.

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[23] See, for instance, Kamien and Schwarts [1982], Grilliches [1984] and Scherer and Ross [1990].

[24] It is, however, possible to incorporate a trade-off between the resources devoted to capital growth and the resources devoted to innovative activities into our model. For instance, the growth parameters γ and/or $-\gamma_0$ in (5) can be made a decreasing function of the innovation coefficient v.

[25] Iwai [1984a, 2000] also develops versions of evolutionary models which assume only the firms currently using the best technology can strike the next innovation. In this case, the process of technological innovations is no longer a Poisson process, so that it is necessary to invoke the so-called "renewal theory" in mathematical probability to analyze the long-run performance of the state of technology.

Hence, the process of technological innovations in the industry as a whole constitute a Poisson process, which is sometimes called the law of rare events. As time goes by, however, innovations take place over and over again, and out of such repetitive occurrence of rare events a certain statistical regularity is expected to emerge.

6. The state of technology in the long-run.

Indeed, not only the process of innovations but also the entire evolutionary process of the state of technology is expected to exhibit a statistical regularity over a long passage of time. To see this, let $\hat{S}_t(c)$ denote the expected value of the cumulative capital share of c at time t. For the purpose of describing the long-run pattern of the industry's state of technology, all we need to do is to follow the path of $\hat{S}_t(c)$. Indeed, it is not hard to deduce from *Hypothesis (IN-a)* the following set of differential equations for $\hat{S}_t(c)$[26]:

$$(18) \quad \dot{\hat{S}}_t(c) = (\gamma\delta+\mu)\,\hat{S}_t(c)(1-\hat{S}_t(c))+\nu(1-\hat{S}_t(c)),$$

for $t \geq T(c)$. It turns out that this is the fourth set of logistic differential equations of this paper, for each of which can be rewritten as $\dot{x} = (\gamma\delta+\mu+\nu)x(1-x)$ with $x \equiv (\hat{S}_t(c)+\nu/(\gamma\delta+\mu))/(1+\nu/(\gamma\delta+\mu))$. It can thus be solved to yield:

$$(19) \quad \hat{S}_t(c) = \frac{1+\dfrac{\nu}{\gamma\delta+\mu}}{1+(\dfrac{\gamma\delta+\mu}{\nu})e^{-(\gamma\delta+\mu+\nu)(t-T(c))}} - \frac{\nu}{\gamma\delta+\mu},$$

for $t \geq T(c)$.[27] .

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[26] The derivation is as follows. Whenever one of the firms with unit costs higher than c succeeds in innovation, the value of $S_t(c)$ increases by the magnitude equal to the innovator's capacity share. ($S_t(c)$ is, however, not affected by the innovation of any of the firms with unit costs c or less, because it only effects an infra-marginal transfer of the capacity share.) As in *note 11*, let $M-M_t(c_i)$ denote the total number of firms with unit costs higher than c_i. The average capacity share of the firms with unit costs higher than c_i is $(1-S_t(c_i))/(M-M_t(c_i))$ and the probability of a successful innovation for each of those $M-M_t(c_i)$ firms is νdt during a small time interval dt. We can then calculate the expected increase in $S_t(c_i)$ due to an innovation as $((1-S_t(c_i))/(M-M(c_i)))(\nu dt)(M-M_t(c_i)) = (\nu dt)(1-S_t(c_i))$. If we divide this by dt and add to it the effects of economic selection and technological imitations given by *(13)*, we obtain *(18)*.

[27] In deducing *(19)*, we have employed a boundary condition: $\hat{S}_{T(c)}(c) = \nu$ or $\hat{S}_{T(c)}(c) = 0$.

Of course, we cannot hope to detect any regularity just by looking at the motion of expected cumulative shares $\hat{S}_t(c)$ given above, for they are constantly pushed to the lower cost direction by recurrent innovations. If, however, we neutralize such declining tendency by measuring all unit costs c relative to the potential unit cost $C(t)$ and observe the relative pattern of the cumulative capital shares, a certain regularity is going to emerge out of seemingly unpredictable vicissitude of the industry's state of technology. Let us thus denote by z the proportional gap between a given unit cost c and the current potential unit cost $C(t)$, or

(20) $z \equiv log c - log C(t).$

We call this variable the 'cost gap' of a given technology at time t. Since the inverse relationship between innovation time $T(c) = -(log c)/\lambda$ and the potential unit cost $C(t) = e^{-\lambda t}$ implies $z = \lambda(t - T(c))$, it is possible to rewrite *(19)* in terms of z as follows:

(21) $\hat{S}_t(c) = \tilde{S}(z) \equiv \dfrac{1 + \dfrac{v}{\gamma\delta + \mu}}{1 + (\dfrac{\gamma\delta + \mu}{v}) e^{-(\gamma\delta + \mu + v)z/\lambda}} - \dfrac{v}{\gamma\delta + \mu}.$

This is the fifth time we have encountered a logistic curve. This time, it represents the 'long-run cumulative distribution' of cost gap z, towards which the relative form of the industry's state of technology has a tendency to approach in the long-run. This distribution is a function only of the cost gap z and is totally independent of calendar time t. *Fig.6* illustrates this distribution.

<center><Insert *Fig.6* around here.></center>

As is seen from *(21)*, the shape of $\tilde{S}(z)$ is determined completely by the basic parameters of our evolutionary model, $\gamma\delta$, μ, v and λ, each representing the force of economic selection, of technological diffusion, of technological innovations and of scientific inventions. (Since both $\gamma\delta$ and μ represent equilibriating forces in our Schumpeterian dynamics, they always appear together in an additive form.) It is not difficult to show that[28]:

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[28] Let $\alpha \equiv v/(\gamma\delta + \mu)$ and $\beta \equiv \lambda/v$. Then, these partial derivatives can be calculated as: $\partial\tilde{S}(z)/\partial(\gamma\delta + \mu)$

$= \mu^{-1}(e^{-\alpha\beta z/(1+\alpha)} - (1 - \alpha\beta z/(1+\alpha)))((\alpha e^{\alpha\beta z/(1+\alpha)})^{1/2} + (\alpha e^{\alpha\beta z/(1+\alpha)})^{-1/2})^{-2} > 0;$

$\partial\tilde{S}(z)/\partial v = (1 - e^{-\alpha\beta z/(1+\alpha)})((\alpha e^{\alpha\beta z/(1+\alpha)})^{1/2} + (\alpha e^{\alpha\beta z/(1+\alpha)})^{-1/2})^{-2} > 0;$

$\partial\tilde{S}(z)/\partial\lambda = -\lambda^{-1}((1+\alpha)z/\alpha\beta)((\alpha e^{\alpha\beta z/(1+\alpha)})^{1/2} + (\alpha e^{\alpha\beta z/(1+\alpha)})^{-1/2})^{-2} < 0.$

(22) $\dfrac{\partial \tilde{S}(z)}{\partial(\gamma\delta+\mu)} > 0,$ $\dfrac{\partial \tilde{S}(z)}{\partial v} > 0,$ and $\dfrac{\partial \tilde{S}(z)}{\partial\lambda} < 0.$

As is illustrated in *Fig.6*, an increase in $\gamma\delta+\mu$, an increase in v, and a decrease in λ shift $\tilde{S}(z)$ counter-clockwise, thus rendering the distribution of efficiencies across firms less disperse than before.

The long-run cumulative distribution $\tilde{S}(z)$ thus deduced is a statistical summary of the way in which a multitude of technologies with diverse cost conditions are dispersed among all the existing capital stocks of the industry. It shows that, while the on-going inventive activities are constantly reducing the potential unit cost, the unit costs of a majority of production methods actually in use lag far behind this potential one. The state of technology therefore has no tendency to approach a classical or neoclassical equilibrium of uniform technology even in the long-run. What it approaches over a long period of time is merely a 'statistical equilibrium of technological disequilibria.'

7. The industry supply curve in the long-run.

Now, the fact that the state of technology retains the features of disequilibrium even in the long-run does have an important implication for the nature of the industry's long-run supply curve. For, as is seen by *(4)*, the relative form of industry supply curve $y_t = Y_t(P_t)b/K_t$ traces the shape of $S_t(c)$, except for the portions of discontinuous jumps. Hence, if the expectation of $S_t(c)$ tends to exhibit a statistical regularity in the form of $\tilde{S}(z)$, the expectation of the relative form of the industry supply curve should also exhibit a statistical regularity in the same long-run form of $\tilde{S}(z)$. Let us denote by p_t the relative gap between a given product price P_t and the potential unit cost $C(t)$, or

(23) $p_t \equiv logP_t - logC(t),$

and call it the 'price gap' at time t. Then, we can obtain the following proposition without paying any extra cost.

Proposition (SC): Under *Hypotheses (CG), (IM'), (PC)* and *(IN-a)*, the expected value of the relative supply curve of the industry $y_t = Y_t(P_t)b/K_t$ will in the long-run approach a functional form of

(24) $\tilde{S}(p_t) \equiv \dfrac{1+\dfrac{v}{\gamma\delta+\mu}}{1+(\dfrac{\gamma\delta+\mu}{v})e^{-(\gamma\delta+\mu+v)p_t/\lambda}} - \dfrac{v}{\gamma\delta+\mu}.$ □

<Insert *Fig.7* around here.>

Fig.7 exhibits the relative form of the industry's long-run supply curve as a function of price gap p. As a matter of fact, it has been drawn simply by turning *Fig.6* around $45°$ line. It therefore moves clockwise as either of $\gamma\delta+\mu$ or ν increases or as λ decreases. This implies that the long-run supply curve becomes flatter, as the joint equilibriating force of economic selection and technological diffusion or the disequilibriating force of creative-cum-destructive innovations becomes stronger, or as the outside force of inventions becomes weaker.

What is most striking about this long-run supply curve, however, is not that it is the "sixth" logistic curve we have encountered in this paper but that it is an *upward-sloping* supply curve!

Let us recall the lower panel of *Fig.1* of the introductory section. It reproduced a typical shape of the long-run supply curve which can be found in any textbook of economics. This horizontal curve was supposed to describe the long-run state of the industry in which the least cost technology is available to every firm in the industry and all the opportunities for positive profits are completely wiped out. However, the relative form of the long-run supply curve we have drawn in *Fig.7* has nothing to do with such traditional picture. There will always be a multitude of diverse technologies with different cost conditions, and the industry supply curve will never lose an upward-sloping tendency, just as in the case of the 'short-run' supply curve of the upper panel of *Fig.1*. There are, therefore, always some firms which are capable of earning positive profits, no matter how competitive the industry is and no matter how long it is run.

We can thus conclude that positive profits are not only the short-run phenomenon but also the long-run phenomenon of our Schumpeterian industry. It is true that the positivity of profits is a symptom of disequilibrium. But, if the industry will approach only a statistical equilibrium of technological disequilibria, it will never stop generating positive profits from within even in the never-never-land of long-run.

8. The determination of the long-run profit rate.

It is one thing to demonstrate the existence of positive profits in the long-run. It is, however, another to analyze the factors which determine the long-run profit rate.

<Insert *Fig.8* around here.>

Let us then look at *Fig.8* which superimposes a demand curve on *Fig.7*. If we suppose that this demand curve is shifting to the right at the same rate as that of the industry's total capital stock and shifting to the bottom at the same rate as that of the potential unit cost, its relative form will become invariant over time. The intersection e^* of such relative demand curve with the long-run relative supply curve then determines the long-run equilibrium price gap p^* and the long-run equilibrium output-capacity ratio $y^* = \tilde{S}(p^*)$. Since we have approximated the profit rate *(Pq-cl)/Pk* of each technology by *b(logP-logc)*, we can also express it as *b((logP-C(t))-(logc-C(t))) = b(p-z)*. This is nothing but the vertical distance between a given price gap and the upward-sloping supply curve. Integrating these individual profit rates from $z = 0$ to $z = p^*$ with their capital shares $\tilde{s}(z) \equiv \tilde{S}'(z)$ as relative weights, we can finally calculate the long-run profit rate r^* of the industry as a whole. Graphically, it can be represented by the shaded area $0e^*y^*$ in *Fig.8*. Algebraically, it can be expressed as[29]:

$$(25) \quad r^* = \int_0^{p^*} b(p*-z)\tilde{s}(z)dz = \frac{(\gamma\delta+\mu+\nu)b}{\lambda}(-\log(1-y^*)-\log(1+\frac{(\gamma\delta+\mu)}{\nu}y^*))$$

$$> 0.$$

We have thus succeeded in giving a complete characterization of the long-run profit rate of our Schumpeterian industry. It is *positive*, indeed.

It is now time to do some comparative dynamics. First, demand effects. It does not require any graphical explanation to see that an upward shift of the relative demand curve raises y^*, because the supply curve is upward-sloping. This of course works to increase the long-run profit rate of the industry r^*. In fact, a differentiation of *(26)* with respect to y^* leads to:

$$(26) \quad \frac{\partial r^*}{\partial y^*} = \frac{(\gamma\delta+\mu+\nu)b}{\lambda}(\frac{1}{1-y^*}+\frac{1}{y^*+\nu/(\gamma\delta+\mu)}) > 0 .$$

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[29] The derivation is as follows. $r^* = b\int_0^{p^*} \tilde{S}(z)dz = b\int_0^{\tilde{S}(p^*)} \tilde{S}(dz/d\tilde{S})d\tilde{S} =$

$b\int_0^{\tilde{S}(p^*)} \tilde{S}(\alpha\beta/((1-\tilde{S})(\alpha+\tilde{S})))d\tilde{S} = b\int_0^{\tilde{S}(p^*)}(\alpha\beta/(1+\alpha))(1/(1-\tilde{S})-\alpha/(\alpha+\tilde{S}))d\tilde{S}$,

where $\alpha \equiv \nu/(\gamma\delta+\mu)$ and $\beta \equiv \lambda/\nu$. We can solve this integral to obtain *(26)*.

A further differentiation of *(26)* **leads to:**

$(27) \quad \dfrac{\partial^2 r^*}{\partial y^{*2}} = \dfrac{(\gamma\delta + \mu + v)b}{\lambda} (\dfrac{1}{(1 - y^*)^2} + \dfrac{1}{(y^* + v/(\gamma\delta + \mu))^2}) > 0$ .c

The industry's long-run profit rate r^* is thus seen to be an increasing and convex function of the equilibrium output-capacity ratio y^*.

This convex relationship between long-run profit rate and output-capital ratio would have a particularly important implication for the dynamic stability, or more appropriately, dynamic instability of our Schumpeterian economy. For *Hypothesis (CG)* immediately implies that the growth rate of fixed investment also becomes on average an increasing and convex function of output-capacity ratio, which is very likely to violate the stability condition for investment-saving equilibrium of the economy as a whole. In the present paper, however, we can only mention this possibility in passing and must resume our comparative dynamics.

Next, let us turn to the supply side and examine the effects of a shift of the long-run supply curve on the industry's long-run profit rate. This, however, turns out to be a far more involved exercise than that on the demand effects, because the results depend on the . I will therefore relegate the detailed discussions to *Appendix* and only summarize the results obtained therein.

When the relative demand curve is perfectly elastic, we have:

$(28) \quad \dfrac{\partial r^*}{\partial(\gamma\delta + \mu)}\Big|_{p^*} = const. \; > 0, \quad \dfrac{\partial r^*}{\partial v}\Big|_{p^*} = const. > 0 \;$ **and** $\; \dfrac{\partial r^*}{\partial \lambda}\Big|_{p^*} = const. \; < 0$.

In this case, an intensification of either the joint equilibriating force of economic selection and technological diffusion or the disequilibriating force of innovations raises the profit rate of the industry in the long-run, whereas an increase in the outside force of inventions reduces it in the long-run.

When, on the other hand, the relative demand curve is absolutely inelastic, we have:

$(30) \quad \dfrac{\partial r^*}{\partial(\gamma\delta + \mu)}\Big|_{y^*} = const. \; > 0 \;$ **for not so small** y^* **and not so large** α,

$\dfrac{\partial r^*}{\partial v}\Big|_{y^*} = const. < 0, \;$ **and** $\; \dfrac{\partial r^*}{\partial \lambda}\Big|_{y^*} = const. \; > 0$.

In this case, as the disequilibriating force of creative-cum-destructive innovations becomes stronger than the equilibriating force of economic selection or swarm-like imitations, or as the average rate of cost reduction of each innovation becomes greater, the industry is expected to generate a

higher profit rate in the long-run. Innovation is not only the source of short-run profits but also the source of long-run profits in an industry with inelastic demand.

Finally, when the relative demand is neither perfectly elastic nor absolutely inelastic, we have:

(31) $\dfrac{\partial \hat{r}^*}{\partial \alpha} > (<) \ 0$ and $\dfrac{\partial \hat{r}^*}{\partial \beta} > (<) \ 0$, when demand curve is inelastic (elastic).

In this general case, as the disequilibriating force of creative-cum-destructive innovations becomes stronger than the equilibriating forces of economic selection and swarm-like imitations, or as the average rate of cost reduction of each innovation becomes greater, the industry is expected to generate a higher profit rate in the long-run, as long as the price-elasticity of demand curve is not so large. However, this tendency will be reversed when the elasticity of industry demand curve becomes sufficiently large.

9. *Pseudo* aggregate production functions.

Since the pioneering works of Solow [1956, 1957], it has become the standard method of neoclassical economics to use the concept of an 'aggregate production function' in accounting the sources of economic growth. It allows economists to decompose variations in GNP into those due to movements along the aggregate production function and those due to shifts of the aggregate production function itself. The former can be attributed to changes in measurable inputs, usually capital and labor, and the latter to changes in technology, an unobservable variable usually inferred from the data as a residual. Early empirical studies of the long-term aggregate growth in advanced capitalist economies found that only a very small portion of the GNP growth can be accounted for by increases in capital and labor, most of the growth being explained by technological progress – an increase in the residual factor. More recent efforts by Maddison [1987] and others, however, have succeeded in reducing the size of the residual factor substantially by incorporating variations in the qualities of capital and labor and other supplementary effects.

The "success" of the neoclassical growth accounting exercises is quite impressive. The challenge to any theory claiming to challenge the

neoclassical orthodoxy is therefore to match its power of tracking down the empirical patterns of the aggregate growth processes of advanced capitalist economies. The most straightforward way to do this is, of course, to set up an empirical study of our own. But in order not to lengthen this already lengthy paper, I choose a short-cut. The purpose of this section is to demonstrate that our evolutionary model is capable of 'calibrating' all the characteristics of neoclassical aggregate production function both in the short-run and in the long-run.[30] If neoclassical growth model is capable of accounting the actual aggregate growth paths of advanced capitalist economies, then our evolutionary model is equally capable of performing the same task. There is no way to differentiate these two models empirically at the macroscopic level. Moreover, our evolutionary model has a decided advantage over the neoclassical model in its ability to integrate microeconomic processes with macroeconomic phenomena. While the neoclassical growth theory simply ignores the complexity of the growth processes we daily observe at the microscopic level, its recognition is the very starting point of our evolutionary model.

Let me begin this 'calibration' exercise by computing the amount of labor employment for each level of product demand. When product demand is small so that price P_t just covers the minimum unit cost c_n, only the first-best technology firms can engage in production and the level of product demand determines that of output Y_t. Because of the fixed proportion technology *(1)*, the level of total employment L_t associated with this output is $c_n Y_t$. When the demand reaches the total capacity of the best technology $k_t(c_n)/b = s_t(c_n)K_t/b$, a further increase in demand is absorbed solely by an increase in P_t, while output is kept at the capacity level. But when P_t reaches c_{n-1}, the second-best technology firms start to produce and all the increase in demand is absorbed by a corresponding increase in output. The relation between output and employment can then be given by $L_t = c_n s_t(c_n)K_t/b + c_{n-1}(Y_t - s_t(c_n)K_t/b)$ until Y_t reaches the total productive capacity of the first- and second-best technology firms $(s_t(c_n)+s_t(c_{n-1}))K_t/b$. In general, the relation between Y_t and L_t can be given by $L_t = \sum_{j=n}^{j=i} c_j s_t(c_j) K_t/b + c_{i-1} (Y_t -$

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[30] In this sense, this section follows up the simulation exercises of Nelson and Winter in [1974] and chap. 9 of [1982].

$\sum_{j=n}^{j=i} s_t(c_j)K_t/b) \equiv \int_0^{ci} c\,dS_t(c)K_t/b + c_{i-1}(Y_t - S_t(c_i)K_t/b)$ whenever $S(c_i)K_t/b \leq Y_t <$

$S(c_{i-1})K_t/b$. **If we divide this relation by** K_t/b **and take its inverse, we can construct a functional relation between the industry-wide labor-capacity ratio** $l_t \equiv L_t b/K_t$ **and the industry-wide output-capacity ratio** $y_t \equiv Y_t b/K_t$ **as:**

(32) $y_t = f_t(l_t)$,

where $l \equiv \int_0^{ci} c\,dS_t(c) + c_{i-1}(f_t(l) - S_t(c_i))$ **whenever** $S_t(c_i) \leq f_t(l) < S_t(c_{i-1})$.

<center><**Insert** *Fig. 9* **around here.**></center>

Fig.9 **depicts this functional relation in a Cartesian diagram which measures labor-capacity ratio** *l* **along horizontal axis and output-capacity ratio** *y* **along a vertical axis. It is evident that this relation satisfies all the properties a neoclassical production function is supposed to satisfy.[31]** *Y* **is linearly homogeneous in** *L* **and** *K*, **because** $y \equiv Yb/K$ **is a function only of** $l \equiv Lb/K$. **Though not smooth, this relation also allows a substitution between** K_t **and** L_t **and satisfies the marginal productivity principle:** $\partial y_t/\partial l_t \leq 1/P_t \leq \partial^+ y_t/\partial l_t$. **(Here,** $1/P_t$ **represents a real wage rate because of our choice of money wage rate as the numeraire, and** $\partial y/\partial l$ **and** $\partial^+ y/\partial l$ **represent left- and right-partial differential, respectively.) Yet, the important point is that this is** *not* **a production function in the proper sense of the word. It is a mere theoretical construct that has little to do with the actual technological conditions of the individual firms working in the industry. As a matter of fact, the technology each firm uses is a Leontief-type fixed proportion technology** *(1)* **which does not allow any capital/labor substitution. It is in this sense that we call the relation** *(32)* **a 'short-run** *pseudo* **aggregate production function,' with an emphasis on the adjective: 'pseudo.'**

The shape of the short-run pseudo production function $y = f_t(l)$ **is determined by a distribution of capital shares** $\{S_t(c_i)\}$ **across technologies. Hence, as this distribution changes, the shape of this short-run function also changes. And in our Schumpeterian industry, the distribution of capital shares is incessantly changing over time as the result of dynamic interplay among capital growth, technological innovation and technological imitation.**

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[31] **See Sato [1975] for the general discussions on the aggregation of micro production functions.**

The most conspicuous feature of the short-run pseudo production function is, therefore, its instability.

In the long-run, however, we know we can detect a certain statistical regularity in the distribution of capital shares $\{S_t(c_i)\}$ out of its seemingly unpredictable movement. We can thus expect to detect a certain statistical regularity in the pseudo production function as well out of its seemingly unpredictable movement. Let $\hat{l}$ and $\hat{y}$ denote the expectation of labor-capacity ratio $l \equiv Lb/K$ and of output-capacity ratio $y \equiv Yb/K$, respectively. Then, we indeed arrive at[32]:

Proposition (PF): Under *Hypotheses (CG), (IM'), (PC)* and *(IN-a)*, the functional relationship between the expected labor-capacity ratio and the expected output-capacity ratio will in the long-run take the form of :

(33) $\hat{y} = \tilde{f}(\hat{l}e^{\lambda t})$,

where the function $\tilde{f}(\cdot)$ is defined implicitly by the following identity:

$$(34) \quad le^{\lambda t} \equiv \int_0^{\tilde{f}(le^{\lambda t})} \left(\frac{\alpha + y}{\alpha(1-y)}\right)^{\frac{\beta\alpha}{1+\alpha}} dy. \qquad \square$$

<Insert *Fig. 10* around here.>

At the seventh time, we have finally graduated from the tyranny of logistic equations! What we have obtained here is a well-behaved function which satisfies all the characteristics a neoclassical production function should have. Indeed, it is not hard to show that $\tilde{f}(0)=0$, $\tilde{f}'(\cdot)>0$, $\tilde{f}''(\cdot)<0$.[33] It is as if total labor force L and total capital stock K produce the total output Y in accordance with an aggregate neoclassical production function $\tilde{f}(\cdot)$ with pure labor augmenting (or Harrod-neutral) technological progress $e^{\lambda t}$. It is, in other words, as if we had entered the Solovian world of neoclassical

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[32] The derivation of this *Proposition* is as follows. Since the short-run 'pseudo' production function *(27)* implies that $l = \int_0^P cdS_t(c)$ whenever $y = S_t(P)$, we have $\hat{l} = \int_0^P cd\hat{S}_t(c)$ whenever $\hat{y} = \hat{S}_t(p)$. But from *(21)* we then have $\hat{l} = \int_0^P cd\tilde{S}(z)$ $= \int_0^p e^{z+\log C(t)}d\tilde{S}(z) = e^{-\lambda t}\int_0^P e^z d\tilde{S}(z) = e^{-\lambda t}\int_0^{\tilde{S}(p)} e^{z(\tilde{S})}d\tilde{S}$ $= e^{-\lambda t}\int_0^{\tilde{S}(p)}((\alpha+\tilde{S})/(\alpha(1-\tilde{S})))^{\alpha\beta/(1+\alpha)}d\tilde{S}$ and $\hat{y} = \hat{S}(p)$. Putting these two relations together, we obtain *(29)*.

[33] More precisely, we have $dy/d(le^{\lambda t}) = ((\alpha+y)/(\alpha(1-y)))^{-\alpha\beta/(1+\alpha)} > 0$ and $d^2y/d(le^{\lambda t})^2 = -\beta((\alpha+y)/(\alpha(1-y)))^{-\alpha\beta/(1+\alpha)-1}/(1-y)^2 < 0$.

economic growth where the economy's growth process could be decomposed into the capital-labor substitution along an aggregate neoclassical production function and the constant outward shift of the aggregate neoclassical production function itself. This is, however, a mere statistical illusion! If we zoomed into the microscopic level of the economy, what we would find is the complex and dynamic interactions among many a firm's capital growth, technological imitation and technological innovation. In fact, as is seen from *(34)*, the functional form of $\tilde{f}(\cdot)$ is a complex amalgam of such basic parameters of our Schumpeterian model as $\gamma\delta$, μ, ν and λ. It is just impossible to disentangle various microscopic forces represented by these parameters and decompose the overall growth process into a movement along a well-defined aggregate production function and an outward shift of the function itself.[34] Indeed, it is not hard to show that both an increase in $\alpha \equiv \nu/(\gamma\delta+\mu)$ and in $\beta \equiv \lambda/\nu$ shift the function $\tilde{f}(\cdot)$ in the downward direction,[35] or

(35) $\quad \dfrac{\partial \tilde{f}(\cdot)}{\partial \alpha} < 0 \quad$ and $\quad \dfrac{\partial \tilde{f}(\cdot)}{\partial \beta} < 0$.

We are after all living in a Schumpeterian world where the incessant reproduction of technological disequilibria prevents the aggregate relation between capital and labor from collapsing into the fixed proportion technology of individual firms. It is, in other words, its non-neoclassical features that give rise to the illusion that the industry is behaving like a neoclassical growth model. It is for this reason we will call the relation *(33)* 'long-run *pseudo* aggregate production function.'

10. Concluding remarks.

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[34] It is true that in the present model the rate of pure labor augmenting technical progress in our pseudo aggregate production function is a given constant λ which is determined exogenously by inventive activities outside of the industry. However, in some of the models presented in a companion paper (Iwai [2000]) this rate becomes also an amalgam of the parameters representing the forces of economic selection, technological diffusion and recurrent innovations.

[35] This can be shown as follows. Let us differentiate (34) (or an equivalent expression given in *note* 32) with respect to α. We then have:

$0 = e^{z(\tilde{S})}\partial\tilde{f}/\partial\alpha + \int_0^{\tilde{S}} e^{z(\tilde{S})}(\partial z(\tilde{S})/\partial\alpha)d\tilde{S}$. Since $z(\tilde{S})$ is an inverse function of $\tilde{S}(z)$ and $\partial\tilde{S}(z)/\partial\alpha < 0$ by (23), we have $\partial z(\tilde{S})/\partial\alpha > 0$. Hence, we have $\partial\tilde{f}/\partial\alpha < 0$ as in (35). We can also show that $\partial\tilde{f}/\partial\beta < 0$ in exactly the same manner.

In the traditional economic theory, whether classical or neoclassical, the long-run state of the economy is an equilibrium state and the long-run profits are equilibrium phenomena. If there is a theory of long-run profits, it must be a theory about the determination of the normal rate of profit.

This paper has challenged this long-held tradition in economics. It has introduced a simple evolutionary model which is capable of analyzing the evolutionary process of the state of technology as a dynamic interplay among many a firm's growth, imitation and innovation activities. And it has demonstrated that what the economy will approach over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but a statistical equilibrium of technological disequilibria which maintains a relative dispersion of efficiencies in a statistically balanced form. Positive profits will never disappear from the economy no matter how long it is run. 'Disequilibrium' theory of 'long-run profits' is by no means a contradiction in terms.

Not only is a disequilibrium theory of long-run profits possible, but it is also 'operational.' Indeed, our evolutionary model has allowed us to calculate (only with pencils and paper) the economy's long-run profit rate as an explicit function of the model's basic parameters representing the microscopic forces of economic selection, technological imitation, recurrent innovation and steady invention. "Without development there is no profit, without profit no development," to quote Joseph Schumpeter once more.[36] The model we have presented in this paper can thus serve as a foundation, or at least as a building block, of the theory of 'long-run development through short-run fluctuations' or 'growth through cycles.' To work out such a theory in more detail is of course an agenda for the future research.

<*Appendix:* Comparative dynamics of supply-side determinants of the long-run profit rate>

<*Appendix:* Comparative dynamics of supply-side determinants of long-run profit rate>

The purpose of this *Appendix* is to deduce *(29)*, *(30)* and *(31)*.

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[36] Schumpeter [1961], p. 154.

Consider first the case of perfectly elastic demand curve. *Fig. A1* juxtaposes a horizontal demand curve on the relative form of a long-run supply curve. We already know from section 8 that an increase in either α or β moves the supply curve counter-clockwise. As is seen from *Fig. A1*, such a supply curve shift transfers the equilibrium point from e^* to e^{**} along the horizontal demand curve and squeezes the long-run profit rate by the magnitude equal to $A \equiv 0e^*e^{**}$. We can easily confirm this graphical exposition by differentiating *(26)* with respect to α and β, keeping p^* constant.

$(A1)$ $\dfrac{\partial r^*}{\partial \alpha}\Big|p^* = const. = b\int_0^{p*} \dfrac{\partial \tilde{S}(z)}{\partial \alpha}dz \equiv -A_\alpha$

$$= -\dfrac{\beta b}{\alpha(1+\alpha)}\int_{e^{-(1+y^*)/(\alpha\beta)}}^1 \dfrac{u-1-\log u}{(1+u)^2}du < 0 ;$$

$\dfrac{\partial r^*}{\partial \beta}\Big|p^* = const. = b\int_0^{p*} \dfrac{\partial \tilde{S}(z)}{\partial \beta}dz \equiv -A_\beta$

$$= -\dfrac{(1+\beta)b}{\alpha\beta}\int_{e^{-(1+y^*)/(\alpha\beta)}}^1 \dfrac{u-1-\log u}{(1+u)^2}du < 0 .$$

This is nothing but *(29)* of the text. Note also that since $\partial\tilde{S}(z)/\partial v > 0$, $\dfrac{\partial r^*}{\partial v}\Big|p^* = const. > 0$.

<Insert *Fig. A1* around here.>

Next, consider the case of absolutely inelastic demand curve. As is shown in *Fig. A2* which juxtaposes a vertical demand curve on the relative form of a long-run supply curve, an increase in either α or β moves the latter counter-clockwise and transfers the equilibrium point from e^* to e^{**} along the vertical demand curve. This raises p^* to p^{**}, while keeping y^* the same as before. The long-run profit rate thus changes from $0e^*p^*$ to $0e^{**}p^{**}$. In order to see whether this amounts to an increase or decrease of r^*, *Fig. A2* decomposes this change of profit rate into two components -- $A \equiv 0e^*e^{**}$ and $B \equiv p^*e^*e^{**}p^{**}$. The first component A represents the "loss" of profit rate due to a universal increase of cost gaps, which corresponds to the profit loss A of the previous case. In the present case of absolutely inelastic demand curve, however, an increase in the long-run equilibrium price gap gives rise to a "gain" of profit rate, as is represented by the second component B. Whether r^* increases or decreases thus depends on whether A is smaller or larger than B. This can be checked by differentiating *(26)* with respect to α and β, keeping y^* constant. We have:

$(A2)$ $\dfrac{\partial r^*}{\partial \alpha}\Big|y^* = const. = b\int_0^{\tilde{S}^{-1}(y*)} \dfrac{\partial \tilde{S}(z)}{\partial \alpha}dz + by^* \dfrac{\partial \tilde{S}^{-1}(y^*)}{\partial \alpha} \equiv -A_\alpha + B_\alpha$

$$= \dfrac{\beta b}{(1+\alpha)^2}((-\log(1-y^*)-\alpha\log(1+\dfrac{y^*}{\alpha}))-(1+\alpha)\alpha(\log(1+\dfrac{y^*}{\alpha})-\dfrac{y^*}{\alpha+y^*})) ;$$

$\dfrac{\partial r^*}{\partial \beta}\Big|y^* = const. = b\int_0^{\tilde{S}^{-1}(y*)} \dfrac{\partial \tilde{S}(z)}{\partial \beta}dz + by^* \dfrac{\partial \tilde{S}^{-1}(y^*)}{\partial \beta} \equiv -A_\beta + B_\beta$

$$= -(\dfrac{\alpha b}{1+\alpha})(\log(1-y^*)+\alpha\log(1+\dfrac{y^*}{\alpha})) > 0 .$$

Although both $-\log(1-y^*)-\alpha\log(1+\dfrac{y^*}{\alpha}) > 0$ and $(1+\alpha)\alpha(\log(1+\dfrac{y^*}{\alpha})-\dfrac{y^*}{\alpha+y^*})$ are

positive in the first expression, the former dominates the latter if we let $\alpha \to 0$. Since $\alpha \equiv v/(\gamma\delta+\mu)$ is assumed to be small, it does not seem unreasonable to suppose the first

expression to be positive. The second expression is always positive. Hence, *(30)* of the main text. Note that we can also calculate $\left.\dfrac{\partial r^{*}}{\partial v}\right|y^{*}=const.$ as

$$\frac{-\alpha^{2}\beta b}{(1+\alpha)^{2}}((-log(1-y^{*})-\frac{\alpha y^{*}}{\alpha+y^{*}})-(log(1+\frac{y^{*}}{\alpha})-\frac{y^{*}}{\alpha+y^{*}})) < 0.$$

<Insert *Fig. A2* around here.>

   Finally, let us consider the general case where industry demand curve is neither perfectly elastic nor absolutely inelastic. As is seen from *Fig. A3*, an increase in either α or β transfers the equilibrium point upward from e^{*} to e^{**} along this downward-sloping demand curve. This raises p^{*} to p^{**} but lowers y^{*} to y^{**}, thereby changing r^{*} from $0e^{*}y^{*}$ to $0e^{**}y^{**}$. We can decompose this change again into $A \equiv 0e^{*}e^{**}$ and $B' \equiv p^{*}e^{*}e^{**}p^{**}$. A represents the "loss" of r^{*} due to a universal increase of cost gaps, and B represents the "gain" due to an increase in the long-run equilibrium price gap. However, B' in *Fig. A3* is not as large as B in *Fig. A2*, for the price elasticity of the demand allows the effect of cost increases to be absorbed not only by price hike but also by quantity reduction. This means that when the demand curve is steeply sloped, the gain B' is likely to outweight the loss A. But, when the demand curve becomes more elastic, B' becomes smaller, and in the limiting case of perfectly elastic demand curve it shrinks to zero.

<Insert *Fig. A3* around here.>

   This graphical explanation can be formalized as follows. First write down the relative form of industry demand function as $y_{t} = \tilde{D}(p_{t})$. Then, p^{*} is determined by the supply-demand equation: $\tilde{S}(p^{*}) = \tilde{D}(p^{*})$. Differentiating this with respect to α and β and rearranging terms, we have: $\partial p^{*}/\partial\alpha = -p^{*}\partial\tilde{S}(p^{*})/\partial\alpha)/(\varepsilon+\eta)$ and $\partial p^{*}/\partial\beta = ((-p^{*}\partial\tilde{S}(p^{*})/\partial\beta)/(\varepsilon+\eta)$, where ε and η are the price-elasticity of the supply curve and of the demand curve, respectively defined as $(\partial\tilde{S}(p)/\partial p)/(\tilde{S}(p)/p)$ and $-(\partial\tilde{D}(p)/\partial p)/(\tilde{D}(p)/p)$. Keeping this in mind and differentiating *(24)*, we obtain:

$$(A3)\quad \left.\frac{\partial r^{*}}{\partial\alpha}\right|\tilde{S}(p^{*})=\tilde{D}(p^{*}) = b\int_{0}^{\tilde{S}^{-1}(y^{*})}\frac{\partial\tilde{S}(z)}{\partial\alpha}dz + bp^{*}\frac{-\partial\tilde{S}(p^{*})}{\partial\alpha}\frac{1}{\eta+\varepsilon} \equiv -A_{\alpha}+B'_{\alpha};$$

$$\left.\frac{\partial r^{*}}{\partial\beta}\right|\tilde{S}(p^{*})=\tilde{D}(p^{*}) = b\int_{0}^{\tilde{S}^{-1}(y^{*})}\frac{\partial\tilde{S}(z)}{\partial\beta}dz + bp^{*}\frac{-\partial\tilde{S}(p^{*})}{\partial\beta}\frac{1}{\eta+\varepsilon} \equiv -A_{\beta}+B'_{\beta}.$$

Note that the component B' in either expression is decreasing in η. In particular, when $\eta = \infty$, B' becomes equal to 0; when $\eta = 0$, B' becomes equal to B in *(A2)*. Hence, we have obtained *(31)* of the text.

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Price

Quantity

0

Price

} normal profit

Quantity

0

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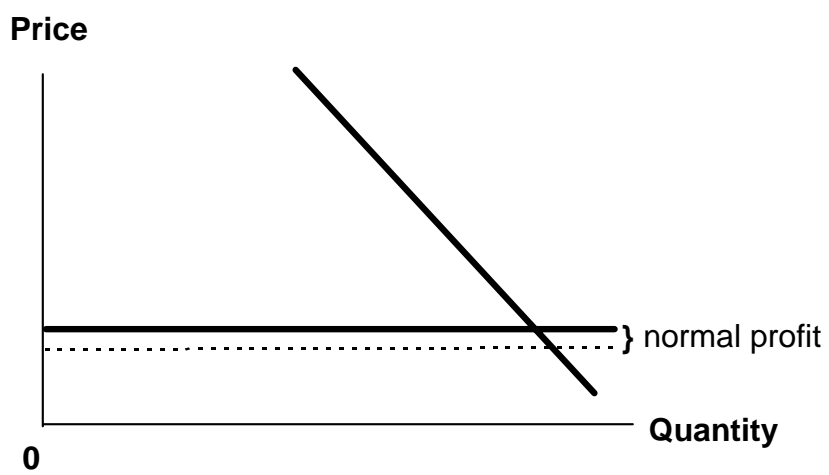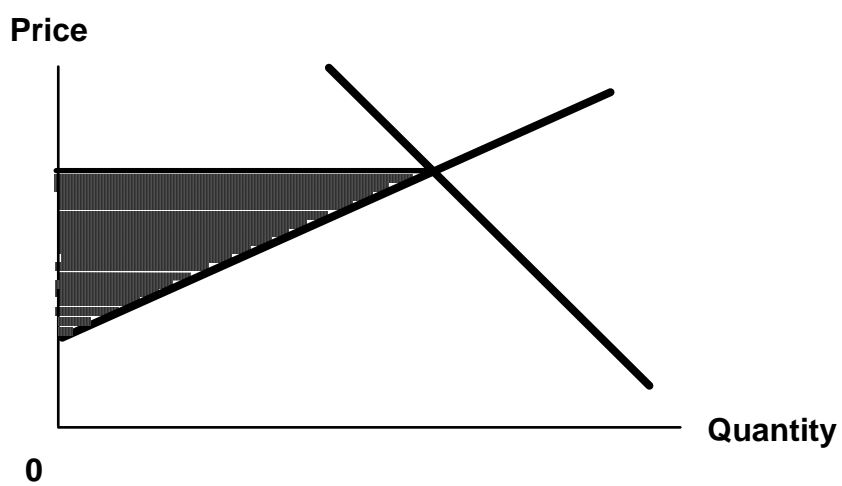
Fig. 1: Industry supply curve in the short-run and in the long-run.
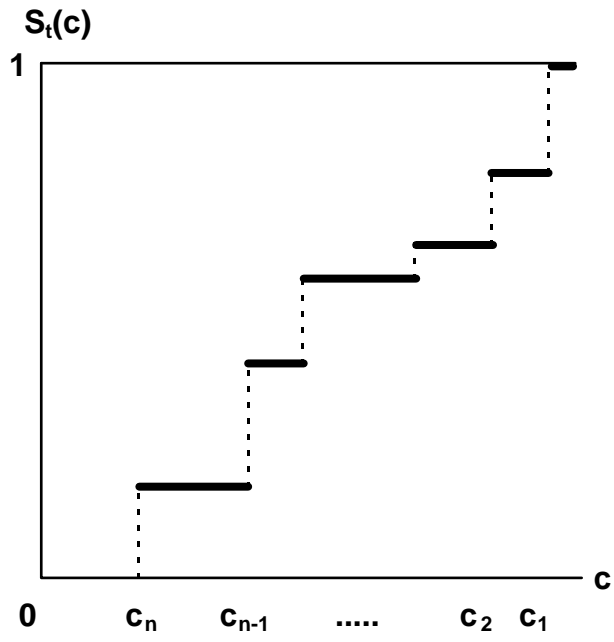
Fig. 2: Cumulative distribution of capacity shares.
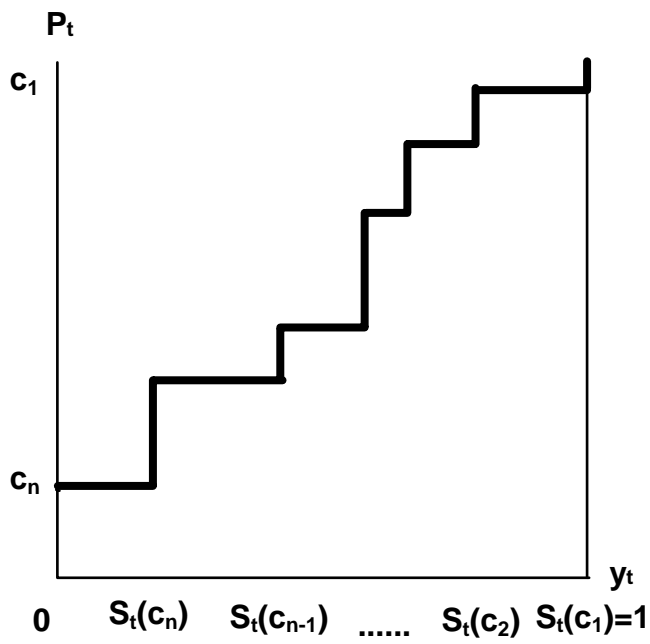
Fig. 3: Relative form of industry supply curve.
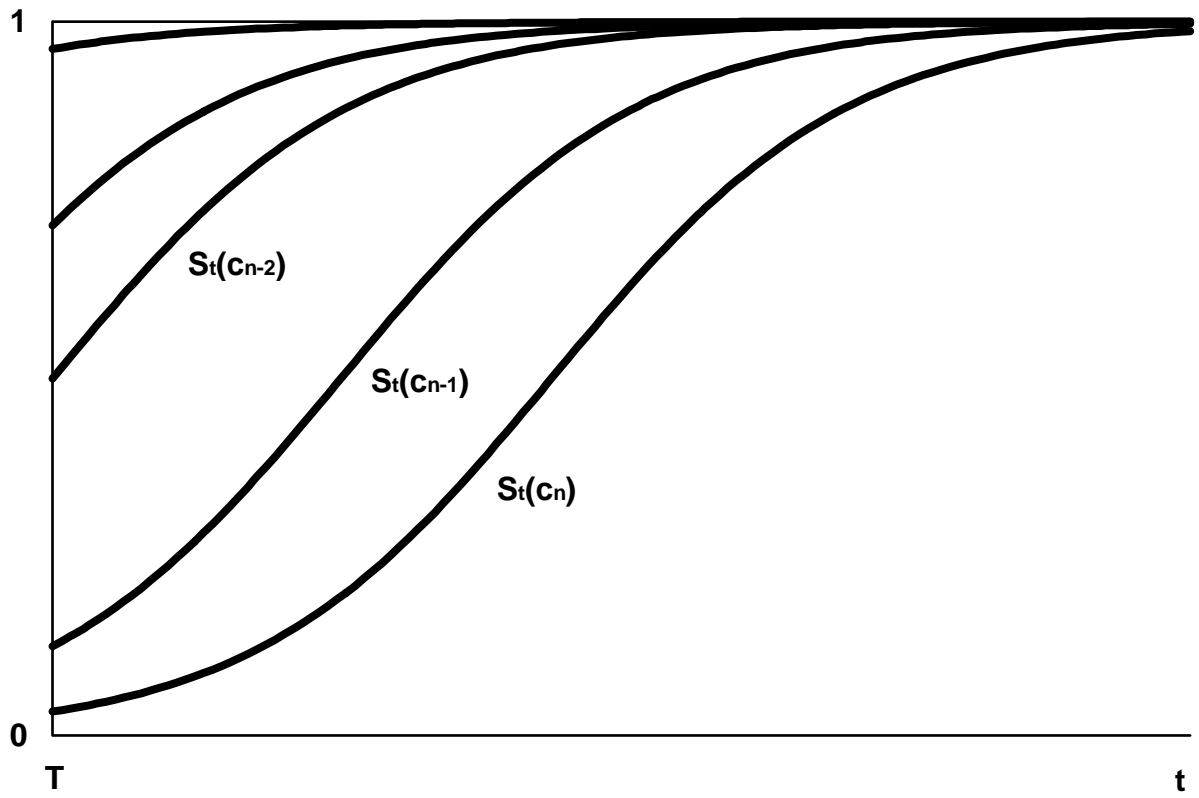
Fig. 4: Evolution of the state of technology under the pressure of either economic selection or technological diffusion.
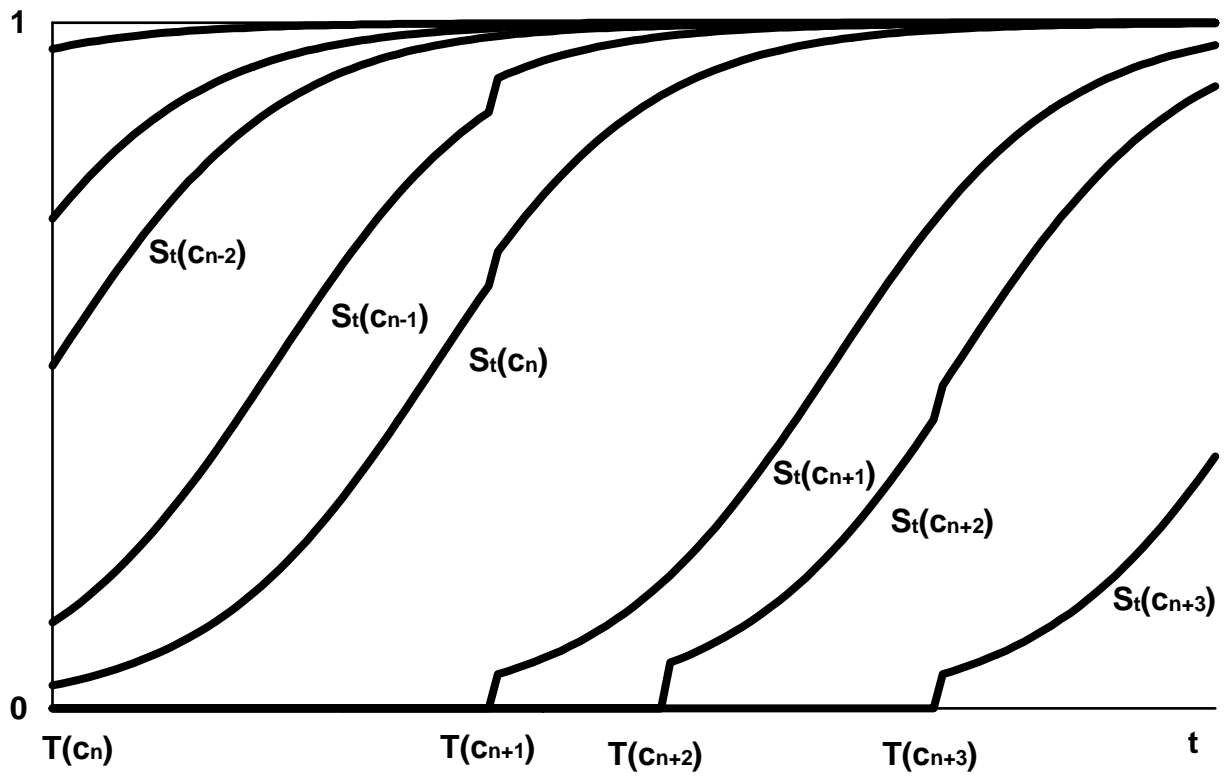
Fig. 5: Evolution of the state of technology under the joint pressure of economic selection,

technological diffusion and recurrent innovations.
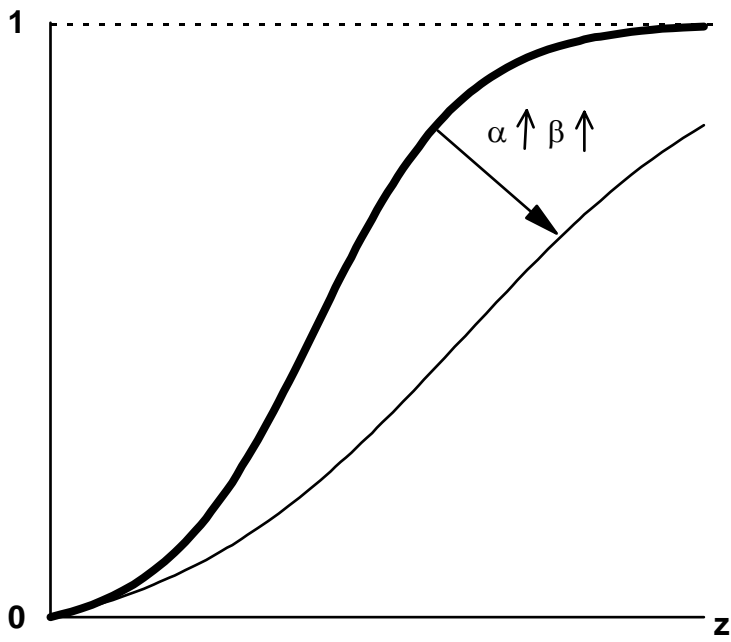
Fig. 6: Long-run cumulative distribution of capacity shares.
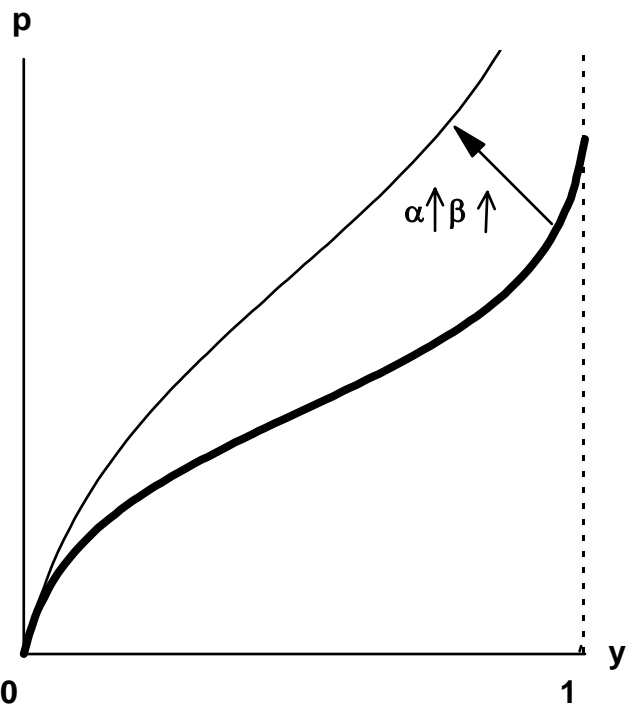
Fig. 7: Long-run industry supply curve.
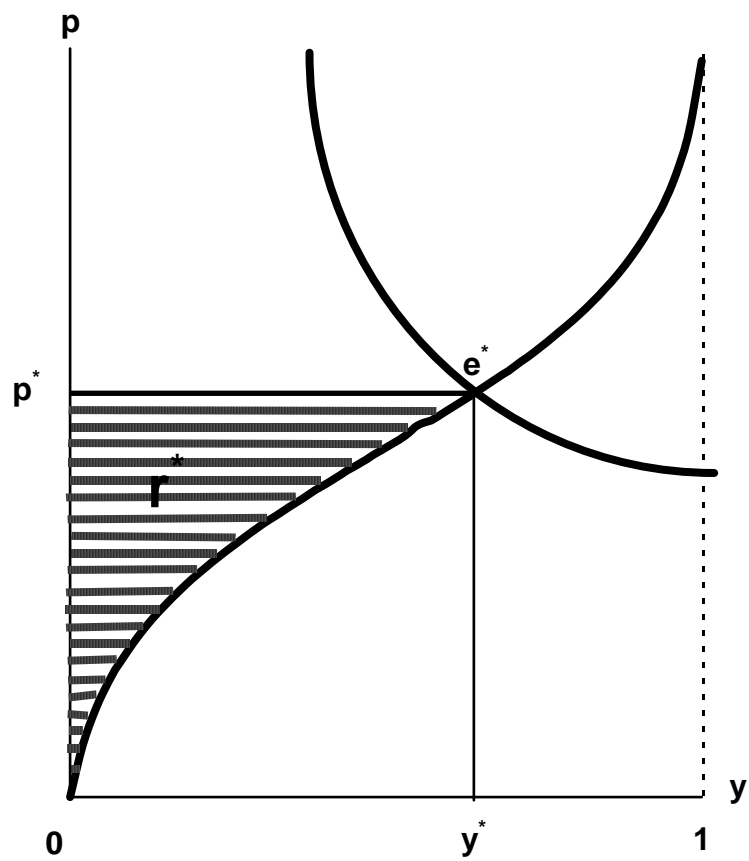
Fig. *8*: Determination of long-run profit rate.
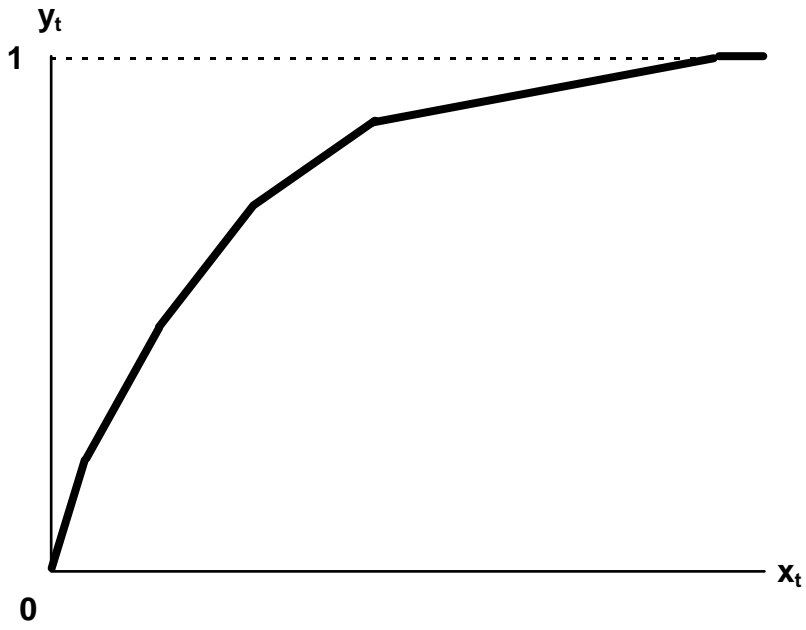
Fig. 9: Short-run 'pseudo' aggregate production function.
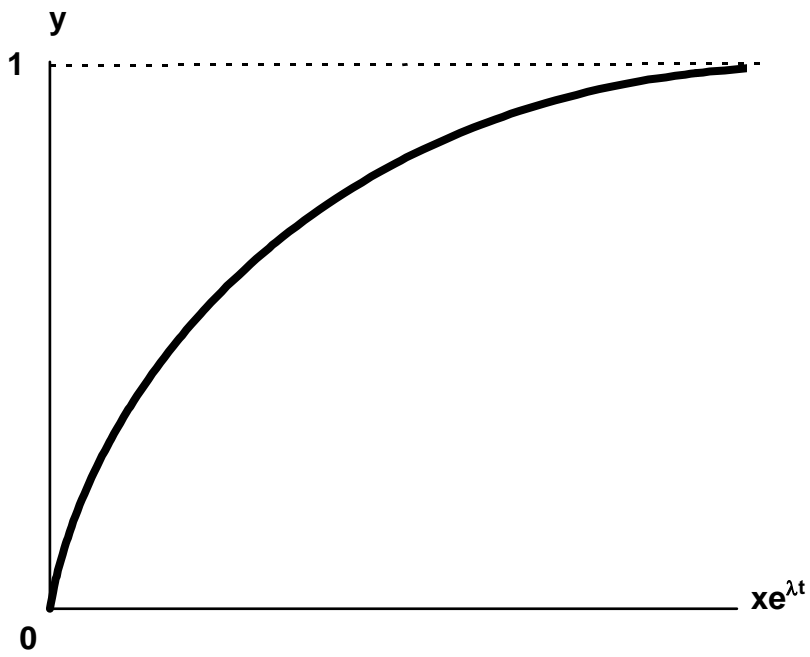
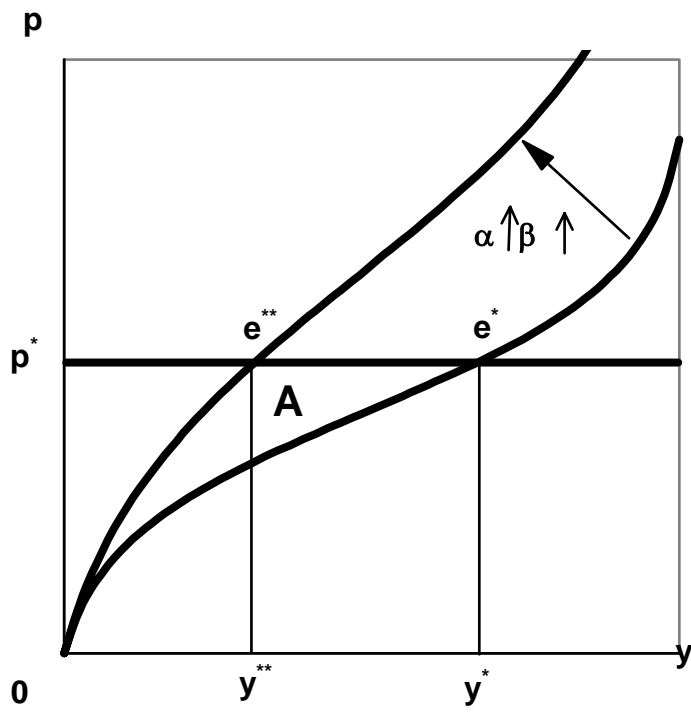*Fig. 10*: Long-run 'pseudo' aggregate production function.

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Fig. A1: The case of perfectly elastic demand curve.

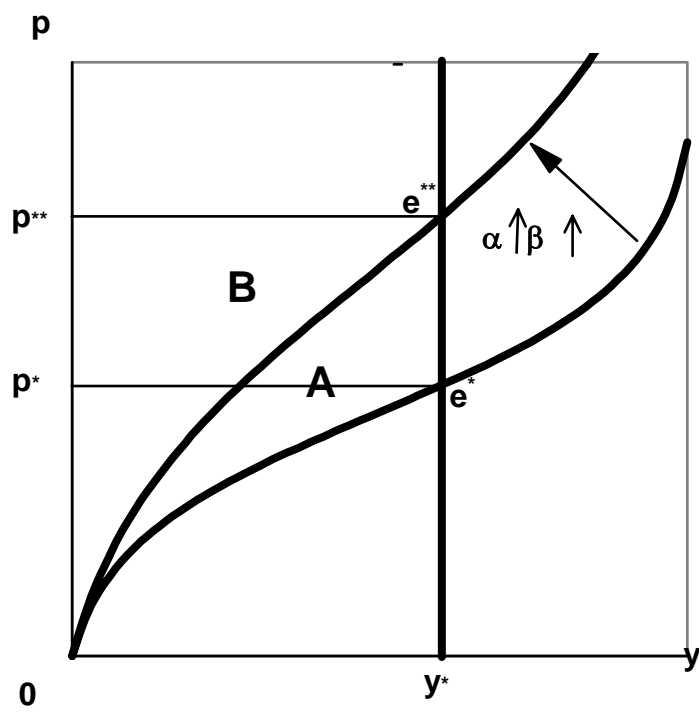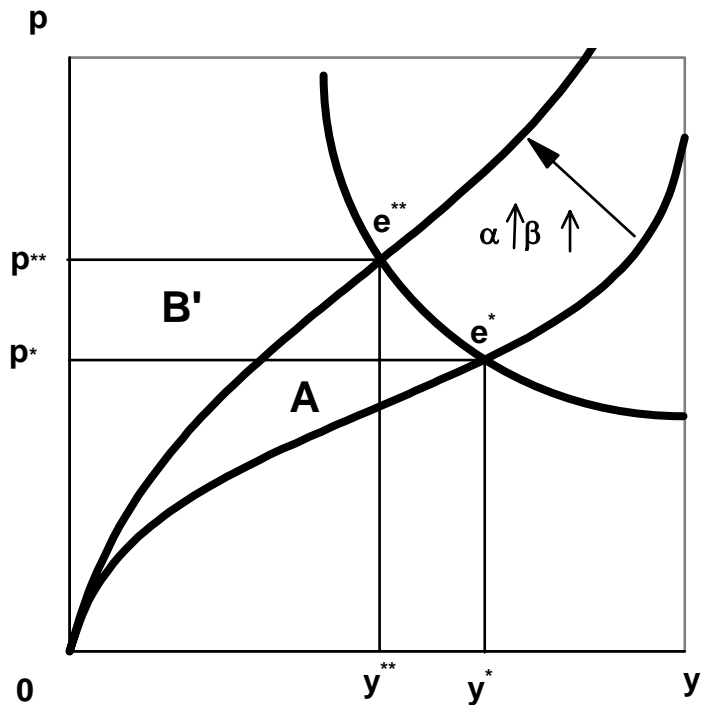*Fig. A2*: The case of absolutely inelastic demand curve.

Fig. A3 : The general case.