Journal of Economic Behavior and Organization 5 (1984) 159–190. North-Holland

SCHUMPETERIAN DYNAMICS

An Evolutionary Model of Innovation and Imitation

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Final version received October 1983

This paper develops a simple evolutionary model of innovation and imitation. It analyzes how dynamic interactions between equilibrating force of imitation and disequilibrating force of innovation mold the evolutionary pattern of an industry's state of technology, and shows that in this Schumpeterian world the industry will never approach a neoclassical equilibrium with perfect knowledge even in the long run. The paper also examines the steady-state efficiency distribution of firms that characterizes the industry's long run and obtains some comparative dynamics results.

1. Introduction

'The essential point to grasp ... in dealing with capitalism' is, according to Joseph Schumpeter (1950, p. 82), that 'we are dealing with an evolutionary process'. The evolutionary character of the capitalist process is due to the fact that 'the fundamental impulse that sets and keeps (its) engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates' (p. 83). Such 'innovation' then creates a market power which enables the innovator to earn a monopoly profit or what is called an entrepreneurial profit, and it is this prospect of gaining entrepreneurial profit that in turn supplies the motives for innovative activities. But the innovator's monoply position is only temporary. As soon as an innovation is made, 'the spell is broken' and the way for others to imitate is opened up. The first innovation draws followers, and then successful imitation again makes it easier for more imitators to follow suit, until finally the innovation becomes familiar and the associated entrepreneurial profit is wiped out, or until the appearance of another innovation renders it obsolete [Schumpeter (1961)]. This process of 'Creative Destruction' — the process that 'incessantly revolutionizes the economic

*I am grateful to Sidney Winter for his discovery of a serious error in the first draft of this paper. Thanks are also due to Neil Ericsson for his valuable comments and to Andrew Joskow for his skillful computational assistance. The remaining errors and inadequacies are, of course, exclusively mine. This research is partly financed by the ministry of education in Japan.

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structure *from within*, incessantly destroying the old one, incessantly creating a new one' — is what Schumpeter regarded as 'the essential fact about capitalism' [Schumpeter (1950, p. 83)].

The orthodox theory of competitive equilibrium consists precisely of assuming this 'fundamental fact about capitalism' *away*. The notion of competitive equilibrium in its most basic form is defined to be a state of affairs in which a set of prices, one for each commodity, balances demand and supply of all commodities and co-ordinates the actions of all market participants who take prices as given and determine demands and supplies accordingly. There is thus no one *within the system* who has any motivation to change the reached position, not to mention the one who strives for creation or destruction. Indeed, from the perspective of the orthodox analysis, the existence of entrepreneurial profit which arises inevitably from successful innovation must be treated as an example of the 'imperfection' of competition; the wave of imitations which relentlessly follows the first success must be classified as an 'externality' to markets; and the entire process of creative destruction is merely an 'adjustment process' which transfers the economy from one equilibrium to another. What Schumpeter considered to be 'the essential fact about capitalism' is regarded here as an aberration from the competitive equilibrium — a slip of the Invisible Hand.

This is the first of a series of papers whose major objective is to develop a simple theoretical framework which is capable of placing the evolutionary process of creative destruction at its central analytical core.[1] It is an attempt to analyze the phenomena of innovation, imitation and growth, not as equilibrium outcomes of the far-sighted choices of optimizing economic agents, but as the dynamic processes moved by complex interactions among individual firms which are constantly striving for survival and growth by their competitive struggle against each other.[2] Indeed, underlying the whole series of papers is a premise that even for the analysis of such 'long-run' economic phenomena it is essential to begin with the study of disequilibrium processes working at the micro level of firms and to trace out carefully the manner in which they interact with each other and cause the aggregate economy to move from one position to the next. Such a 'disequilibrium' view of technological change and economic development has certainly been foreign to the orthodox economists who tend to identify 'long run' with 'equilibrium' and dismiss 'disequilibrium' as mere 'short-run' problems.[2a]

[1]For recent attempts at formalizing the 'vision' of schumpeter, see Winter (1969), Nelson and Winter (1982) and Futia (1980). Our indebtedness to their works ought to be obvious.

[2]Our Schumpeterian dynamics should therefore be distinguished from the so-called neo-Schumpeterian models of Scherer (1967), Kamien and Schwartz (1972, 1975), Loury (1979), Dasgupta and Stiglitz (1980a, 1980b) and others. Their analyses treat the firm's R&D activity as a one-shot game and fail to situate it in a long-run evolutionary process of industrial development. We, however, intend to incorporate the firms' long-term decisions on innovation and imitation policies in our future studies.

[2a]See Iwai (1981) for another attempt to introduce 'disequilibrium' view into economics.

2. The state of technology

No one fails to notice a wide gap between the industrial structure we observe in the real economy and the idealized world of neoclassical economics in which all firms are supposed to have free access to the most efficient technological knowledge. Fig. 1 exhibits how the ratio of payroll to value added (a good index of the reciprocal of labor productivity) is distributed across establishments in metal stampings industry (SIC no. 3461) in 1958 and 1963. Establishments with a remarkably wide range of productivities co-exist in an industry, and this wide dispersion of productivities has no tendency to disappear over time. The state of technology in this industry appears to be in perpetual disequilibrium. In fact, the metal stampings industry is chosen as an example (almost) arbitrarily among over four hundred industries classified by SIC, and the similar pattern can be discerned in most other industries.[3]

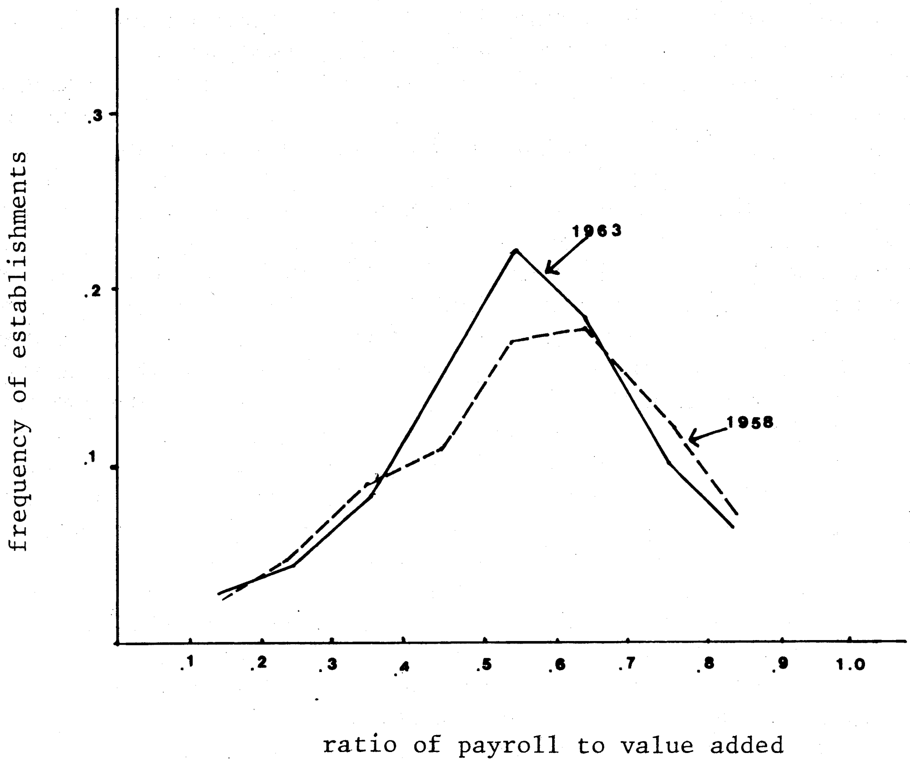

Fig. 1. The frequency distribution of efficiency in metal stampings industry. [*Source*: U.S. Department of Commerce (1968).]

[3]See U.S. Department of Commerce (1968). Sato (1975) analyzed the efficiency distributions reported in this census in detail. He also found similar patterns in Japanese cotton-spinning industry and Norwegian fish-food products and non-electrical machinery industries.

We start from this simple (but by no means the only possible) observation that the state of technology in most industries appears to be in a state of perpetual disequilibrium. And one of the aims of this paper is to develop, with the guidance of Schumpeterian vision, a simple mathematical model which is capable of demonstrating that the state of technology will be forever in disequilibrium.

Consider an industry which consists of a large number of firms competing with each other. Some firms are taking active part in the workings of the industry by turning out products; others may be passive participants that are not engaged in production at the moment but ready to start it when the right time comes. Some of the firms are leaving the industry for good, while some new firms are making an entry into it. In the present paper, however, we are not concerned with the firms' production decisions nor with the process of turnover of firms. Instead, in order to lay out the basic framework of our Schumpeterian dynamics as simple as possible, we shall abstract from the former and ignore the latter in what follows. (See, however, section 8.)

We shall denote by M total number of firms which participate either actively or passively, in the working of a given industry. M is assumed to be constant over time.

A firm may be producing a product that is homogeneous throughout the industry, or a unique product of its own which is differentiated from the products of others, depending upon the structure of a particular industry in question. In fact, we shall present, in this series of papers, a theoretical framework which is capable of dealing with any of these alternative industry structures. Unless there is a universal access to the same and best technology, production method actually employed is different from firm to firm. Let us identify each production method by a positive real number c. Although we call this number the firm's 'unit cost' (in terms of numeraire) for the sake of concreteness, it is only one of many possible interpretations. All that is needed in most of our subsequent investigations is a convention that the smaller the value of c is, the more profitable is the corresponding production method. If the number of production methods co-existing in an industry is finite (n) we can represent them by a list of unit costs, $c_n < c_{n-1} < \cdots < c_i < \cdots < c_1$, arranged in ascending order. The first in the list c_n then designates the unit cost of the best practice method and the last c_1 the unit cost of the worst production method. To describe the 'state of technology' of an industry at a point in time, it is therefore necessary to stipulate how these different production methods are distributed across firms.

Let $f_t(c)$ represent the relative frequency of firms whose unit cost equals c at time t. It is, in other words, the frequency function of unit costs at time t. Since only the production methods with unit costs $c_1, c_2, \ldots, c_n$ are actually employed at time t, the value of $f_t(c)$ is zero for any other value of unit cost. [By convention we have $f_t(c_1) + \cdots + f_t(c_n) = 1$.] Let $F_t(c)$ represent the relative frequency of firms whose unit costs are equal to *c or less* at time t. It is, in other words, the cumulative frequency function of unit costs at time t. Needless to say, $F_t(c)$ can be formally defined as

$$F_t(c) \equiv f_t(c_i) + f_t(c_{i+1}) + \cdots + f_t(c_n) \tag{1}$$

for $c_{i-1} < c \leqq c_i$. [We set, as convention, $F_t(c) = 0$ for $c < c_n$ and $F_t(c) = 1$ for $c \geqq c_1$.] Fig. 2 illustrates the relation between $f_t(c)$ and $F_t(c)$.

The frequency function $f_t(c)$ or alternatively, the cumulative frequency function $F_t(c)$ represents how a variety of production methods from the most profitable to the least, are distributed across firms at a given point in time. Either of them gives us a snapshot picture of the industry's 'state of technology' at a given point in time. Unlike the paradigm of the orthodox economics, however, the state of technology is not a given datum to the industry. As time goes by and future unfolds itself, dynamic competition among firms for technological superiority constantly changes it from one configuration to another. The state of technology is never static and never exogenous in a capitalist economy.

The main aim of the following sections is to describe how the state of technology evolves over time in a Schumpeterian industry.

3. Imitation or diffusion process of technology

In the world of Schumpeterian competition, each firm is constantly striving for a better production method. There are basically two means by which that aim can be achieved. A firm may succeed in putting a new production method into practice by its own R&D effect; i.e., it may succeed in 'innovation'. The firm can also direct its eyes towards outside; it may indeed 'imitate' one of the more profitable methods which are currently employed by other firms. The evolution of the state of technology is therefore determined by the interaction of these two dynamic forces. In order to give an orderly exposition of this complex evolutionary process, however, we shall devote the present section exclusively to the study of the process of imitation, postponing that of the process of innovation until the next section.

Schumpeter wrote:

> '[T]he carrying out of new combinations is difficult.... However, if one or a few have advanced with success, many of the difficulties disappear. Others can then follow these pioneers, as they will clearly do under the stimulus of the success now attainable. Their success again makes it easier, through the increasingly complete removal of the obstacles..., for

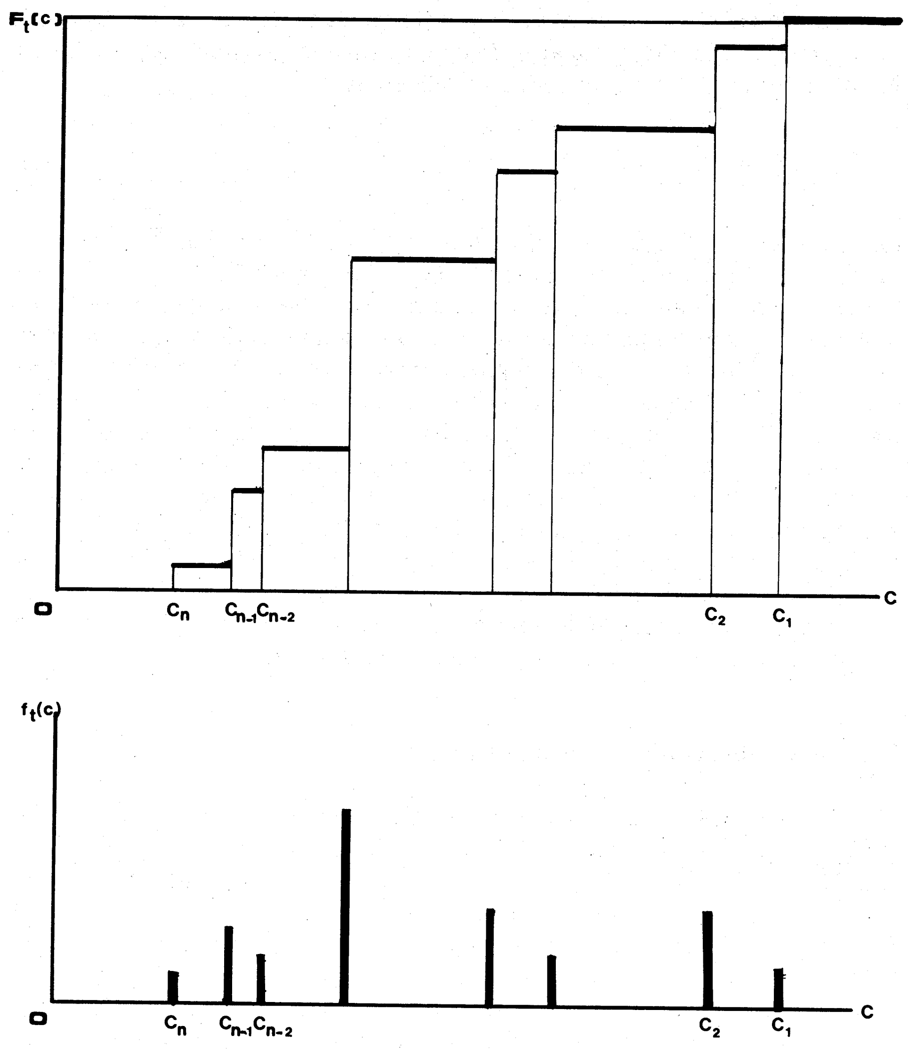

Fig. 2. The relation between $F_t(c)$ and $f_t(c)$.

more people to follow suit, until finally the innovation becomes familiar and the acceptance of it a matter of free choice.' [Schumpeter (1961, p. 228).]

For our purposes it is, however, necessary to translate this somewhat picturesque description of the process of imitation into a much more prosaic mathematical language. Indeed, there exist several alternative models which can do this, but the particular one chosen in this paper is characterized by the following extremely simple hypothesis:

Hypothesis (*IM'*). The probability that a firm is able to copy a particular production method is proportional to the frequency of firms which employ that method in the period in question. The firm, of course, implements only the method whose unit cost is lower than the one currently used by it.[4]

Formally, it will be assumed that the probability that a firm of unit cost c_i imitates a production method of unit cost c during a small time interval between t and $t+\Delta t$ is equal to

$$\mu f_t(c)\Delta t \quad \text{for} \quad c<c_i$$

and

$$0 \qquad \text{for} \quad c \geqq c_i,$$

where $\mu>0$ is a parameter which summarizes the effectiveness of the firm's imitation activity.

The value of the imitation parameter μ should be influenced by the particular imitation policy the firm has come to adopt in its long-run pursuit of survival and growth. Indeed, in recent years a small but growing body of literature has been concerned with empirically indentifying factors which influence the value of the imitation parameter or something of the kind. [See, for instance, Mansfield (1968, ch. 8), Davies (1979), Mansfield, Schwartz and Wagner (1981) and papers quoted therein.] The present paper, however, is not concerned with the analysis of how each firm shapes up its imitation policy and chooses (or at least influences) the value of the imitation parameter μ. The main objective here is rather to work out formally the dynamic mechanism through which a given long-run imitation policy of the firms (along with a given long-run innovation policy, to be discussed in sections 6 and 7) structures the evolutionary pattern of the industry's state of technology. We shall, therefore, assume in this paper that the imitation parameter μ is a given constant whose value is a legacy from the past. We shall also assume that the value of μ depends neither on the current unit cost of the firm nor on the unit cost of the production method it wishes to imitate.[5] We shall assume further, for the sake of simplicity, that a new

[4]In the next paper which takes an explicit account of the process of capacity growth, this hypothesis will be modified into *Hypothesis* (*IM*): The probability that a firm is able to copy a particular production method is proportional to the share of total productive capacity which employs that method in the period in question.

[5]It is, however, possible to replace this assumption by another: that the firm imitates only the best practice production method, or $\mu f_t(c)\Delta t$, for $c=c_n$ (the unit cost of the best practice method), and 0 otherwise, and then to reproduce qualitatively most of the results obtained under this. We intend to report our analysis under this alternative assumption in a forthcoming paper. Yet another alternative specification would be that the firm is able to imitate the production method whose efficiency is one rank above the one it is currently using, or the probability that a firm with a unit cost c_i imitates a production method of unit cost c is $\mu f_t(c)\Delta t$, for $c=c_{i+1}$, and 0 otherwise. Unfortunately, this alternative model has so far resisted our analysis.

production method once copied can be implemented to the entire productive capacity within a firm without any cost and without any delay. Indeed, throughout this series of papers, all technical changes are supposed to be of the disembodied type. The problem of intra-firm diffusion process of new technical knowledge [as is analyzed, for instance, by Mansfield (1968, ch. 9)] is thus set aside from our investigation.

Now, under hypothesis (IM′) it is possible to analyze the evolutionary pattern of the industry's state of technology in the following simple manner. Consider the way in which $F_t(c_i)$, the relative frequency of firms with unit cost c_i or less, changes its value from time t to $t+\Delta t$. It is clear that this relative frequency increases whenever one of the firms whose unit cost is higher than c_i succeeds in imitating one of the firms with unit cost c_i or less. [Of course, even among firms whose unit costs are lower than c_i, the relatively higher cost firms are imitating the production methods of the lower cost firms. It is, however, plain that these intra-marginal imitation activities result only in intra-marginal transfers of frequencies and do not affect the value of $F_t(c_i)$ itself.] Now the relative frequency of firms whose unit costs are higher than c_i equals $f_t(c_{i-1})+f_t(c_{i-2})+\cdots+f_t(c_1)$, which can be conveniently rewritten as $1-F_t(c_i)$ by (1). On the other hand, hypothesis (IM′) tells us that the probability that *each* of these firms succeeds in imitating one of the production methods with unit cost c_i or lower during a time interval between t and $t+\Delta t$ is equal to $\mu f_t(c_i)\Delta t+\mu f_t(c_{i+1})\Delta t+\cdots+\mu f_t(c_n)\Delta t$, which can be conveniently rewritten as $\mu F_t(c_i)\Delta t$ by (1). (Here, the probability that a firm succeeds in copying two or more production methods simultaneously can be ignored so long as the time interval Δt is small.) We can therefore compute the *expected* increase in $F_t(c_i)$ during a time interval between t and $t+\Delta t$ as the product of these two expressions: $\{\mu F_t(c_i)\Delta t\}\cdot\{1-F_t(c_i)\}$. In fact, if the total number of firms M is very large, the so-called law of large numbers allows us to use this expression for a good approximation for the *actual* increase in $F_t(c_i)$. In what follows, we assume this is indeed the case and treat the above expressions as representing the actual increase in $F_t(c_i)$.[6]

We have thus obtained an equation which describes the change in the relative frequency of firms of unit cost c_i or less, from time t to $t+\Delta t$, effected by the firms' imitation activities in an industry:

$$F_{t+\Delta t}(c_i)-F_t(c_i)=\mu F_t(c_i)(1-F_t(c_i))\Delta t. \tag{2}$$

Furthermore, if we divide the both sides of this equation by Δt and let Δt approach zero, we can transform it in the following differential equation:

[6]If M is not large enough, what follows can be interpreted as the analysis of the 'expected' behavior of the state of technology. Note further that the analysis of the long-run average behavior of the state of technology to be given in sections 6 and 7 is independent of the largeness of M.

$$\dot{F}_t(c_i) = \mu F_t(c_i)(1 - F_t(c_i)), \tag{2'}$$

where $\dot{F}_t(c_i)$ represents the time derivative of $F_t(c_i)$. Since the same argument can be applied without any modification to any value of unit cost, we have, in fact, obtained the following series of differential equations:

$$\begin{aligned} \dot{F}_t(c_n) &= \mu F_t(c_n)(1 - F_t(c_n)), \\ &\vdots \\ \dot{F}_t(c_i) &= \mu F_t(c_i)(1 - F_t(c_i)), \\ &\vdots \\ \dot{F}_t(c_1) &= \mu F_t(c_1)(1 - F_t(c_1)). \end{aligned} \tag{3}$$

It requires only a moment's reflection to recognize that each of the above differential equations is nothing but a well-known 'logistic differential equation', which appears frequently in population biology and mathematical ecology. [See, for example, Pearl and Reed (1924), Lotka (1925), or any modern textbook on these subjects. Samuelson (1947, pp. 291–294), also contains a useful discussion on this form of differential equation.] It is very easy to show that this logistic differential equation has the following form of explicit solution, which is called the 'logistic growth curve':

$$F_t(c_i) = 1/[1 + (1/F_T(c_i) - 1)\exp[-\mu(t-T)]], \qquad i = 1, 2, \dots, n, \tag{4}$$

where $\exp(\cdot)$ stands for exponential, and $F_T(c_i)$ represents the cumulative frequency function at a given time $T(\leq t)$ in the past.

Fig. 3 illustrates the foregoing result. Each of the S-shaped curves traces a logistic growth curve that represents the growth pattern of the cumulative frequency function of firms. In particular, the one at the lowest layer depicts the growth pattern of the relative frequency of firms with the least unit cost c_n. When only a small number of firms employ this production method, its growth is hesitant and slow. But as this number gradually increases, imitation activities of the less efficient firms become more and more successful. 'The spell is broken', and a bandwagon sets in motion. The growth rate then accelerates, until a half of total population comes to adopt this method. Once this median point is passed, the effect of saturation steps in and the growth rate starts decelerating. But the growth itself continues until the whole population of firms is swamped by this best practice method. The fate of the less efficient production method, on the other hand, can be easily read by tracing out the changing width of a strip formed by two adjacent logistic curves. Initially its number may expand by absorbing the firms with less efficient techniques. But sooner or later it will lose ground to the more efficient techniques, and will find its way to the eventual extinction.

The idea of using the logistic curve to describe a bandwagon phenomenon that can be commonly observed in a variety of diffusion process of a new

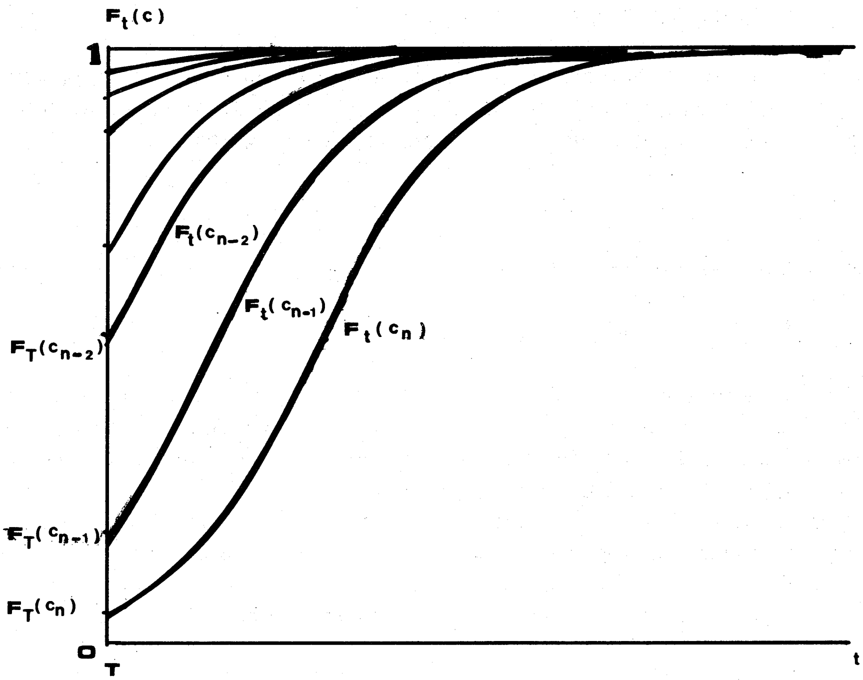

Fig. 3. The evolution of the state of technology under the pressure of imitation process.

idea, new technique, new instrument, and so forth is not new. Indeed, there is an abundant literature on this application in economics and other social sciences. [See, for example, Coleman, Katz and Menzel (1957), Griliches (1957), Ozga (1960) and Mansfield (1968, ch. 8).] Outside of social sciences the so-called 'models of epidemics' deal with mathematically similar problems. [See, e.g., Bailey (1957).] What seems novel about our foregoing analysis is its application of the logistic law to the description of the evolutionary pattern of the whole array of production methods co-existing side by side at the same time. And it is this small innovation which allows us to study the dynamic interaction between processes of imitation and innovation in an integrated manner, as we shall soon see.

4. Innovation

As is shown in the preceding section, firms' imitation activities will gradually upgrade their production techniques, and, if other things are equal, all the firms will eventually succeed in adopting the best practice method. This limiting state must be the paradigm of neoclassical economics in which every market participant is supposed to have complete access to the best technical knowledge of the society. Other things, however, do not forever remain the same. The tendency towards technological uniformity among firms is bound to be upset by a sudden introduction of a new and better production method by one of the firms. Indeed, to destroy the stalemate brought about by the imitation process and to create a new industrial structure is the role our capitalist economy has assigned to Schumpeterian entrepreneurs or to innovative firms. It is this 'process of creative destruction' that is 'what capitalism consists in and what every capitalist concern has got to live in' [Schumpeter (1950, p. 83)]. Let us now turn to the formal analysis of this perennial gale of creative destruction.

Suppose that at some point in time one of the firms finally succeeds in implementing a new production method whose unit cost equals $c_{n+1}(<c_n)$. We denote by $T(c_{n+1})$ the time at which this method is introduced for the first time and call it 'the innovation time' for the production method with unit cost c_{n+1}. (This somewhat clumsy notation will make more sense in section 6.) Since the total number of firms is M and hence each firm's share is $1/M$, this innovation creates a new relative frequency of the magnitude of $1/M$ at the new and lower unit cost c_{n+1}. That is, we have

$$F_{T(c_{n+1})}(c_{n+1})=1/M. \tag{5}$$

No sooner does this innovation occur than do all the other firms start struggling to imitate it. A firm or two will eventually make a headway, and a wave of imitation will then follow. Under hypothesis (IM′), this sets in motion a new logistic growth curve of $F_t(c_{n+1})$ from the initial condition (5) given above. Hence, we have for $t \geqq T(c_{n+1})$

$$F_t(c_{n+1})=1/[1+(M-1)\exp[-\mu(t-T(c_{n+1}))]]. \tag{6}$$

How does this innovation affect the evolutionary pattern of the state of technology of the industry as a whole? The answer to this question depends upon whether or not the innovator has used the best practice production method before innovation. We first examine a special case.

Let us suppose that the innovator of c_{n+1} has employed the then best practice method c_n before the innovation time $T(c_{n+1})$. In this case, the size of $f_t(c_n)$ declines by $1/M$ at the time of $T(c_{n+1})$, but this decline is recouped at the same time by the new creation of an equal magnitude of $f_t(c_{n+1})$, as shown in (5). Obviously, this exchange of an equal mass of frequency leaves unaffected the cumulative frequency $F_t(c_n)$, for it is nothing but the sum of $f_t(c_n)$ and $f_t(c_{n+1})$. It then follows that even after the innovation time $T(c_{n+1})$, the cumulative frequency $F_t(c_n)$ keeps moving along the same old logistic growth curve (4). Indeed, since the innovation in question involves no other production method, all the other cumulative frequencies must follow the same old logistic curves as well. Part of fig. 4 around the innovation time

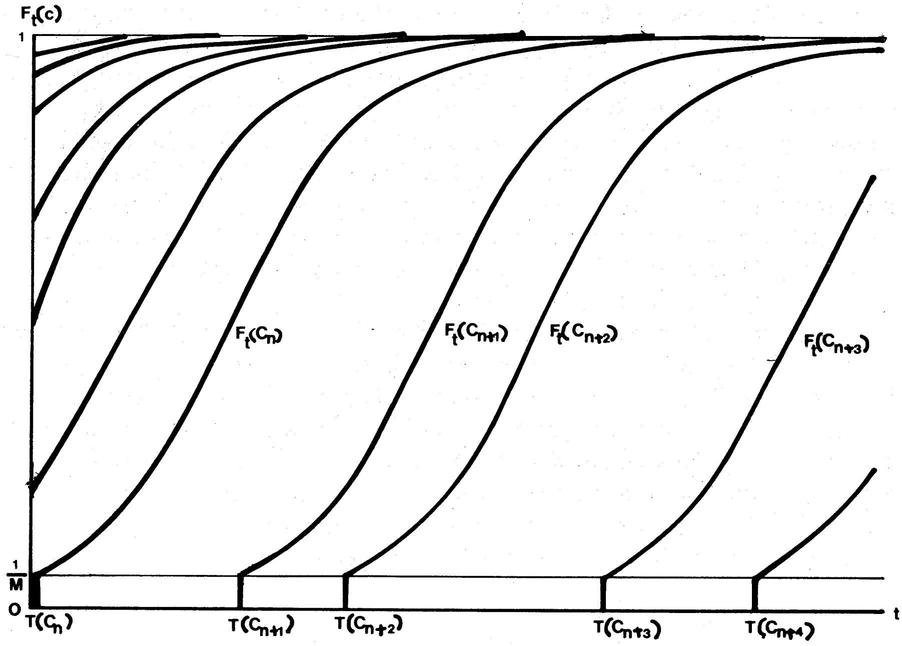

Fig. 4. The evolution of the state of technology under the joint pressure of innovation and imitation — the case where only the technologically most advanced firms can innovate.

$T(c_{n+1})$ illustrates all this. By comparing it with fig. 3, the reader can immediately see that the only alteration we made to the latter is to superimpose a new logistic growth curve that starts with an initial mass $1/M$ at time $T(c_{n+1})$.

Innovation is not a single-shot phenomenon. No sooner than an innovation occurs, a new round of competition for a better production method begins. And no sooner than a winner of this game is named, another round of competition for a still better production method is set out. And so forth. Innovation is by nature a recurrent process.

Accordingly, let $T(c_{n+2})$, $T(c_{n+3}),\ldots,T(c_N),\ldots,$ denote times at which production methods with unit costs $c_{n+2}>c_{n+3}>\cdots>c_N>\ldots$ are introduced for the first time into an industry, respectively. We call $T(c_N)$ the 'innovation time' of the production method with unit cost c_N and $T(c_N)-T(c_{N-1})$ the 'waiting time' for a new method with c_N. (There is, of course, no reason to believe that these innovation times are evenly distributed over time.) Then, at each innovation time $T(c_N)$ a new cumulative frequency $F_t(c_N)$ starts its logistic growth path from the (suddenly created) initial frequency $1/M$.

If, as in the case of the first innovation, innovations always emerge from the class of firms which have practiced the then best production method, we can repeatedly apply the same argument as was given earlier and claim that none of these successive innovations perturb the logistic growth patterns of all the cumulative frequencies of the currently practiced production methods. They only add a new logistic growth curve one by one from the bottom at each innovation time. This process is explained in fig. 4.

We are now in a position to examine the more general case by removing the supposition so far made that innovations always emerge from the class of technologically most advanced firms. Then, the evolutionary pattern of the state of technology becomes slightly more complex.

Suppose, at time $T(c_{n+1})$, new production method with unit cost c_{n+1} is introduced by a firm whose pre-innovation unit cost was not the smallest in the industry but was equal to a somewhat outdated value, say c_{n-2}. Just as before, this innovation sets out a logistic growth process of the cumulative frequency of $F_t(c_{n+1})$ from a newly created initial frequency $1/M$. [See eq. (6).] But, unlike the previous special case, growth paths of some of the already existing cumulative frequencies have to undergo a certain adjustment at the time of innovation $T(c_{n+1})$. For instance, $F_t(c_{n-1})$, the cumulative frequency of c_{n-1}, which was the sum of $f_t(c_n)$ and $f_t(c_{n-1})$ before the innovation, has now to add to itself a frequency $f_t(c_{n+1})$ whose value suddenly jumps from zero to $1/M$. Similarly, $F_t(c_n)$ experiences a discrete jump in its value by $1/M$. Both $F_t(c_n)$ and $F_t(c_{n-1})$ then resume their logistic growth paths from these adjusted values from then on. The rest of the cumulative frequencies, $F_t(c_{n-2}), F_t(c_{n-3}), \ldots$ remain unperturbed by the innovation of c_{n+1} and keep the same logistic growth paths even after that. For, in the case of these cumulative frequencies, the emergence of the new frequency $f_t(c_{n+1})$ is offset by the decline of the frequency $f_t(c_{n-2})$ by the same magnitude $1/M$. The set of logistic growth paths thus began at $T(c_{n+1})$ will continue until some of them are again upset by another innovation at the next innovation time $T(c_{n+2})$. At time $T(c_{n+2})$, yet another set of logistic growth paths will be set off only to be upset once again at the next innovation time $T(c_{n+3})$. And so forth. Fig. 5 presents an evolutionary pattern of the state of technology in this general case.

5. A specific model of innovation

In the preceding sections we have seen how the process of imitation and the process of innovation interact with each other and mold the evolutionary history of an industry's state of technology. The process of imitation works essentially as an equilibrating force that continually but slowly tends the industry towards a static equilibrium, in which all firms employ the same and best production technique available. The function of innovation, on the other hand, lies precisely in upsetting such an equilibrating tendency. It is a disequilibrating force which breaks up the existing order of an industry and forces the state of technology to become more progressive but more volatile.

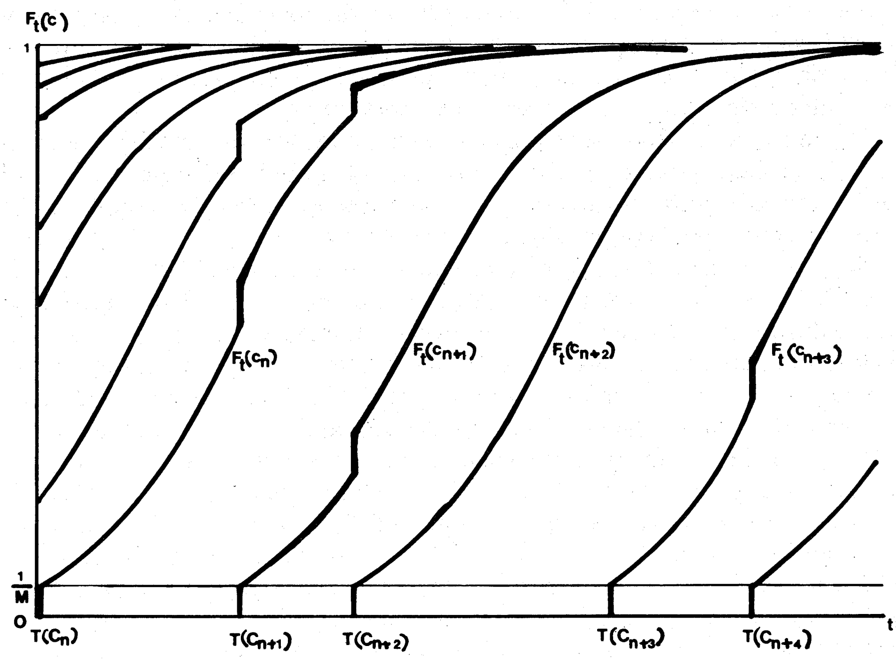

Fig. 5. The evolution of the state of technology under the joint pressure of innovation and imitation — the general case.

The purpose of this section and the next two is to study how the dynamic interaction of these opposite forces will determine the course of the development of the state of technology in the long run. To this end we have to specify the structure of firms' innovation activities in more detail.

Basic or applied scientific researches in private firms, governmental institutions and academia, weekend experiments of amateur inventors in their backyard garages, and so forth continuously expand the stock of technical knowledge potentially applicable to industrial production. But such a continuous inflow of new technical knowledge or 'inventions' does not necessarily lead to a corresponding improvement of production methods actually employed in an industry. 'As long as they are not carried into practice, inventions are economically irrelevant' [Schumpeter (1961, p. 88)]. For the purpose of industrial production, the potentiality must be transformed into the actuality; a production method hitherto untried must be put into industrial practice. This is what we mean by the word 'innovation', which must be conceptually distinguished from 'invention'.[7]

[7]This Schumpeterian dichotomy is, of course, a much oversimplified conceptualization of the inherently complex process of technical activities of modern corporations. We shall stick to this scheme solely for the sake of formalization.

Let us denote by $C(t)$ the unit cost of the best production method that is 'technologically possible' at time t but has thus far resisted the actual use in the industry. (For the sake of simplicity we ignore all the problems associated with the uncertainty as well as fuzziness inherent in delineating what is technologically possible from what is not.) We call $C(t)$ the unit cost of the potential production method or, more simply, the 'potential unit cost' at time t. It is then reasonable to suppose that the continuous inflow of technological knowledge or continuous supply of inventions constantly reduces the potential unit cost of the industry, so that we have

$$\dot{C}(t) < 0. \tag{7}$$

This paper, however, does not probe into the mechanism of inventive activity itself; inventions are supposed to occur outside the industry and beyond the control of the individual firms. This is, of course, a heroic assumption to maintain.

It then becomes possible to characterize the notion of 'innovation' formally as the activity by which a firm puts into practice the potential production method and thus succeeds in reducing its unit cost to the level of potential unit cost. Now, let $T(c)$ denote the inverse function of $C(t)$, defined by

$$T(C(t)) \equiv t \quad \text{or} \quad C(T(c)) \equiv c. \tag{8}$$

[Because of the monotone decreasingness of $C(T)$ with respect to T, as is assumed in (7), $T(c)$ is also a monotonically decreasing function of c.] We know that if an innovation occurs at time t it introduces a production method with $C(t)$ unit cost for the first time into an industry. It then follows that if a particular production method with unit cost c is presently in use it must have been introduced into the industry at time $T(c)$, for in view of the inverse relation (8) we have $c = C(T(c))$. The function $T(c)$ can then be interpreted as the 'innovation time' for a given production method with unit cost c, and this interpretation and notation are perfectly consistent with the definition of the same concept we introduced in the preceding section.

Later we shall find it useful to introduce the following hypothesis which further specializes the dynamics of the potential unit cost $C(t)$.

Hypothesis (PC). The potential unit cost is declining at a constant rate over time.

More formally this hypothesis supposes that

$$C(t) = \exp(-\lambda t), \tag{7'}$$

where λ is a positive constant. [For convenience, we set $C(0) = 1$.] Under this special hypothesis, the innovation time $T(c)$ — the inverse of $C(t)$ — can be expressed simply as a logarithmic function of c, or

$$T(c) = -\frac{1}{\lambda} \ln c. \tag{8'}$$

This special hypothesis will simplify our later exposition.

We have seen above what innovation consists of. But we have not seen who does innovation. It is now the time to specify in more detail the process that characterizes the way innovation occurs. We shall indeed consider two alternative models, which can be regarded as two polar cases spanning more realistic situations as their convex combinations. Let us explore these two models separately.

6. The state of technology in the long run (I)

In the first case, we postulate the following hypothesis concerning the stochastic nature of innovative activity.

Hypothesis (IN-a). Every firm has a small but equal chance for successful innovation at every point in time.

Let $v \cdot \Delta t$ be the probability that a firm succeeds in carrying out an innovation during a small time interval Δt; where v is a positive constant which is supposed to be of the much smaller order of magnitude than the imitation parameter μ. Then, the probability that an innovation is successfully carried out by *one of* the firms during a time interval Δt becomes equal to

$$vM\Delta t. \tag{9}$$

The probability that two or more firms simultaneously succeed in innovation is extremely small and hence ignored. Hypothesis (IN-a) amounts to saying that the occurrence of innovation is subject to the law of rare events or to the Poisson law which supposes that whether or not an innovation occurs in any time interval is independent of whether or not an innovation occurs in any time interval preceding it. (This is called the lack of memory property of the Poisson process.)

The innovation parameter v represents the effectiveness of each firm's innovation activity. Its value should, therefore, reflect a particular innovation policy the firm has come to choose as a critical pillar of its long-run growth strategy. In the present paper, however, we are concerned only with analyzing how the evolutionary pattern of the industry's state of technology is causally determined by a given innovation policy of the firm, together with its imitation policy. The study of how the firm selects a particular innovation policy in the long run and how this long-run decision process reflects the evolutionary pattern of the industry's state of technology is left for the future research.[8]

The state of technology of an industry is a historical outcome of the dynamic interaction between the process of imitation and the process of innovation. The process of imitation is an equilibrating force which moves the entire state of technology along the family of logistic growth curves, whereas the process of innovation is a disequilibrating force which disturbs this smooth journey and restructures the state of technology from time to time. As time goes on, however, innovation takes place over and over again. After a long period of time, it is expected that a certain statistical regularity will emerge out of this random pattern of the occurrence of innovations. [For instance, it is not difficult to show that after a long passage of time the average rate of innovation tends to approach a constant value νM, under the Poisson hypothesis (IN-a).] Indeed, not only the dynamic pattern of innovation but the entire state of technology is also expected in the long run to exhibit a tendency towards certain statistical regularity as a long-run averaging result of the dynamic balance between the forces of imitation and of innovation.

Let $F_t^*(c)$ represent the *expected* cumulative frequency function of unit costs at time t. We shall now turn to the study of the behavior of this expected cumulative frequency function. Since we are concerned only with describing the industry's state of technology 'in the long run', this is all that we have to do.

Now, we know from (3) that the cumulative frequency function $F_t(c)$ *increases by* $\mu F_t(c)(1-F_t(c))\Delta t$, if no innovation occurs during a time interval, $[t, t+\Delta t]$. If, on the other hand, an innovation occurs during the same time interval, it creates a new cumulative frequency $F_t(C(t))$ of the size equal to $1/M$. When the innovator has belonged to the class of firms whose unit costs are higher than c, it automatically raises the value of $F_t(c)$ by the same magnitude $1/M$ in addition to the effect of imitation $\mu F_t(c)(1-F_t(c))\Delta t$. When, however, the innovator is from the class of firms whose unit costs are less than or equal to c, the innovation effects only an intra-marginal exchange of an equal mass of frequency and leaves the value of $F_t(c)$ unaffected. Since by hypothesis (IN-a) the probability of an innovation during a time period of Δt is $\nu M\Delta t$ and the fraction of firms whose unit costs are higher than c is $1-F_t(c)$, the expected number of innovators whose unit costs are higher than c can be calculated as $\nu M(1-F_t(c))\Delta t$ during $[t, t+\Delta t]$.

[8]Empirical literature on the factors which influence the innovation policy of firms is enormous. Excellent surveys are Kamien and Schwartz (1975 and 1982; ch. 3), and Scherer (1979, ch. 15).

We can thus conclude that the cumulative frequency function $F_t(c)$ increases *on average* by $\{\mu F_t(c)(1-F_t(c))+\nu M(1-F_t(c))(1/M)\}\Delta t$ from time t to $t+\Delta t$. In terms of expected cumulative frequency function $F_t^*(c)$, we can state this result as

$$\dot{F}_t^*(c)=\mu F_t^*(c)(1-F_t^*(c))+\nu(1-F_t^*(c)). \tag{10}$$

This is indeed a logistic differential equation of $F_t^*(c)+\mu/\nu$, which has an explicit solution of the form

$$F_t^*(c)+\frac{\nu}{\mu}$$
$$=(1+\nu/\mu)\Bigg/\left(1+\left[\frac{1+\nu/\mu}{F_{T(c)}^*(c)+\nu/\mu}\right]\exp\left[-(\mu+\nu)(t-T(c))\right]\right) \tag{11}$$

for $t\geqq T(c)$; where $T(c)$ is the innovation time for a given unit cost c and $F_{T(c)}^*(c)$ is the expected value of the cumulative distribution at that point of time. Although $F_{T(c)}(c)$ equals $1/M$ when an innovation occurs, the probability of an innovation at a particular instant is of course equal to zero. Hence, $F_{T(c)}^*(c)=0$, which also implies $\dot{F}_{T(c)}^*(c)=\nu$, and we can simplify (11) as

$$F_t^*(c)=\frac{1+\nu/\mu}{1+(\mu/\nu)\exp\left[-(\mu+\nu)(t-T(c))\right]}-\frac{\nu}{\mu}. \tag{11'}$$

In order to proceed further, it is necessary at this point to keep in mind the obvious fact that unit costs of firms in the industry has a tendency to decline over time under the joint pressure of the forces of imitation and innovation. It is therefore futile to expect that the shape of the expected cumulative frequency function itself will exhibit a tendency towards any statistical regularity. If there exists any statistical regularity at all, it must be of the form which is relative to the declining tendency of the unit costs as a whole in the industry. In order to capture this relative nature of the possible statistical regularity, let us introduce the following new variable:

$$z\equiv\ln c-\ln C(t). \tag{12}$$

This variable measures how the unit cost c of a given production method is proportionally in excess of the potential unit cost $C(t)$ prevailing at time t. We call this the 'cost gap', for short. By definition, the value of cost gap becomes zero for the production method which has been just innovated; it takes a positive value for any other production method actually in use. As we shall see shortly, the use of this new measure of efficiency will enable us to neutralize the declining tendency of unit costs.

Indeed, if we recall the logarithmic relation (8′) between a unit cost c and its innovation time $T(c)$, the gap between t and $T(c)$ in eq. (11′) can be rewritten in terms of the cost gap as z/λ. Hence, we obtain a relation

$$F_t^*(c) = \tilde{F}(z) \equiv \frac{1+\nu/\mu}{1+(\mu/\nu)\cdot\exp[-(\mu+\nu)z/\lambda]} - \frac{\nu}{\mu}, \tag{13}$$

which is independent of the calendar time t! $\tilde{F}(z)$ above is the 'long-run average cumulative frequency function' of the cost gap we have sought to deduce under the hypothesis (IN-a). It is a function only of the cost gap, $z \equiv \ln c - \ln C(t)$, and not of the value of the potential unit cost $C(t)$ itself. Fig. 6 illustrates the structure of this long-run average cumulative frequency function. It has the shape of a truncated logistic growth curve, with growth parameter $(\mu+\nu)/\lambda$ and initial slope ν/λ.

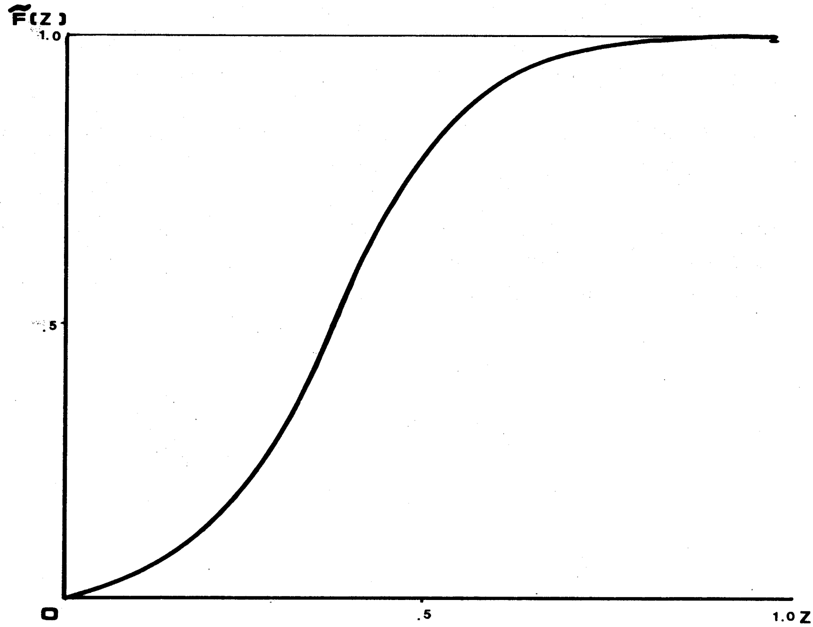

Fig. 6. The long-run average cumulative frequency function of cost gaps under hypothesis (IN-i), when $\lambda=0.05$, $\nu=0.01$ and $\mu=0.50$.

The long-run average cumulative frequency distribution obtained above is a long-run statistical summary of the way in which firms in the industry are distributed over a multitude of diverse production methods with different unit costs. It shows that while the continuous inflow of new technological knowledge constantly reduces the potential unit cost over time, the industry will never be able to enjoy its fruits fully and unit costs of a majority of firms will always lag behind the potential one. The industry's state of technology will thus never approach a neoclassical equilibrium of uniform technological knowledge. Indeed, it is only the relative shape of the distribution of firms over different cost conditions which exhibits any tendency for a statistical regularity over the long-run course of events in the industry.

The dynamic interaction between the forces of innovation and imitation, together with the exogenous inflow of new technological knowledge, is what maintains the relative configuration of the state of technology in a statistical equilibrium in the form of (13). In order to study how a change in each of these forces will shift this delicate statistical balance, it is easier to examine the density form of the long-run average frequency function, given by

$$f(z) \equiv \frac{\mathrm{d}\tilde{F}(z)}{\mathrm{d}z}$$

$$= \frac{(\mu+\nu)^2}{\lambda\mu} \Bigg/ \left[\sqrt{\frac{\nu}{\mu}} \cdot \exp\left(\frac{\mu+\nu}{2\lambda} z\right) + \sqrt{\frac{\mu}{\nu}} \cdot \exp\left(-\frac{\mu+\nu}{2\lambda} z\right)\right]^2 \tag{14}$$

for $z \geqq 0$. As is depicted in fig. 7, the long-run average density distribution is a smooth bell-shaped curve, truncated at the left. It has a peak of the height equal to $(\mu+\nu)^2/4\lambda\mu$ at the value of cost gap equal to $[\lambda/(\mu+\nu)] \cdot \ln(\mu/\nu)$, and the intercept equal to ν/λ at the zero cost gap. It is thus not difficult to see that an increase in the declining rate of potential unit cost, λ, tends to widen the cost gaps of the industry and at the same time disperse their distribution

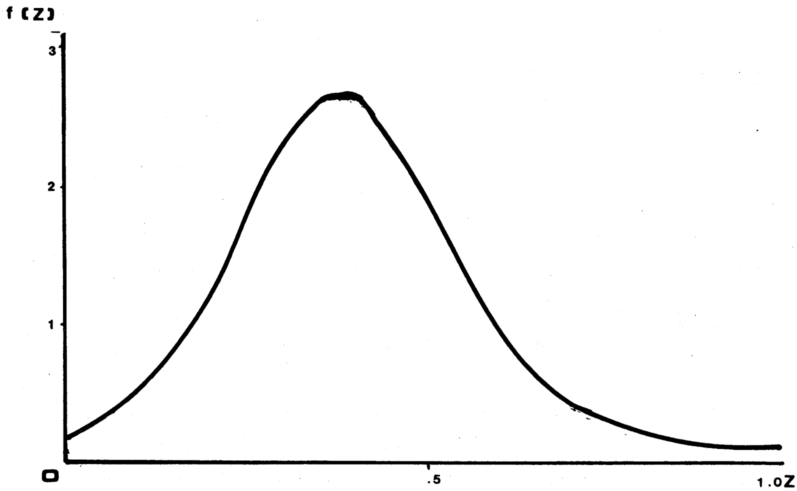

Fig. 7. The long-run average density function of cost gaps under hypothesis (IN-i).

across firms; that an increase in the rate of innovation, ν, tends, albeit weakly, to narrow the cost gaps and concentrate their distribution; and that an increase in the rate of imitation, μ, also tends to narrow the cost gaps and concentrate their distribution.[9] Figs. 8, 9 and 10, respectively, illustrate these comparative statics results numerically. [The base values of parameters, $\lambda=0.05$, $\nu=0.01$, $\mu=0.50$ mean, if the number of firms in the industry is 20, (i) that the potential unit cost declines 5% annually, (ii) that the average lag between invention and innovation is 5 years, and (iii) that it takes on average 5.89 years for half of the firms to succeed in imitating an innovation.]

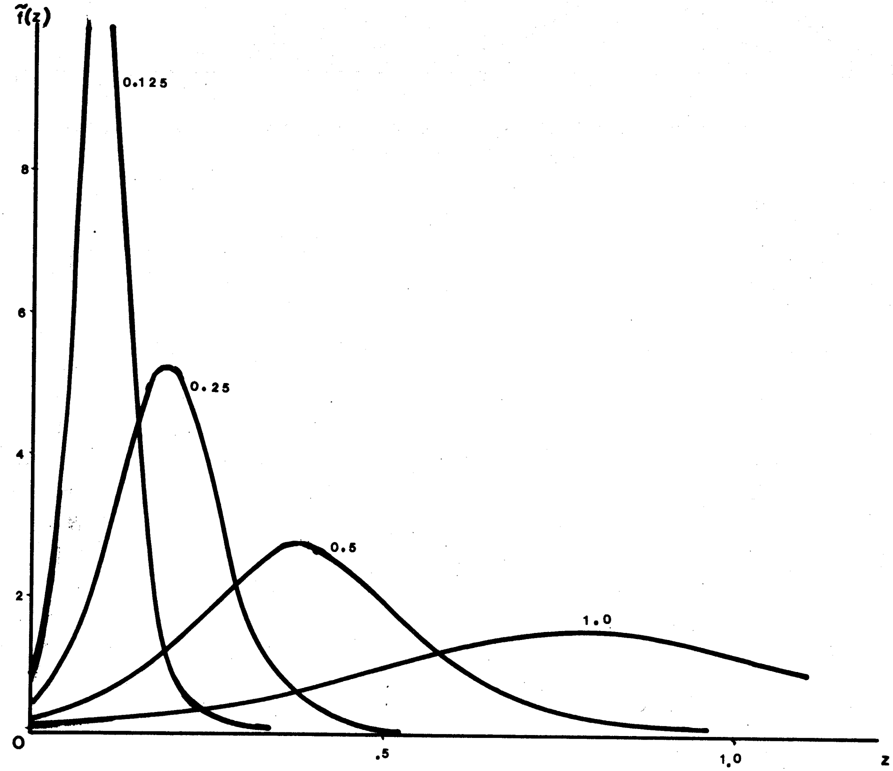

Fig. 8. The long-run average density functions under hypothesis (IN-i) for various values of λ (where ν and μ are fixed at 0.01 and 0.50 respectively).

Note in passing that, while the occurrence of an innovation disrupts the existing order of the industry and makes its state of technology more disperse than before, an increase in its probability tends to increase the technological efficiency of the industry as a whole in the long run. This apparent conflict between short-run effect and long-run consequence of

[9]An increase in μ may widen the average cost gap if $1+\nu/\mu<\ln(\mu/\nu)$. But this somewhat perverse case can be ignored as long as ν is sufficiently small relative to μ.

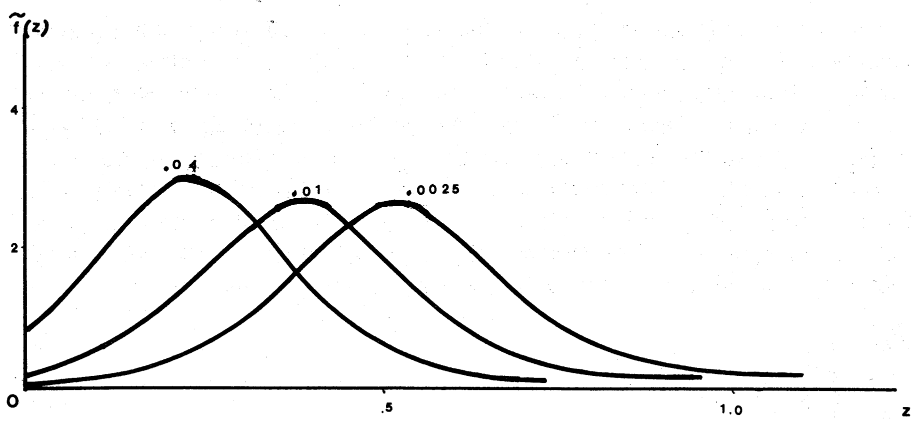

Fig. 9. The long-run average density functions under hypothesis (IN-i) for various values of ν (where $\lambda=0.05$ and $\mu=0.10$).

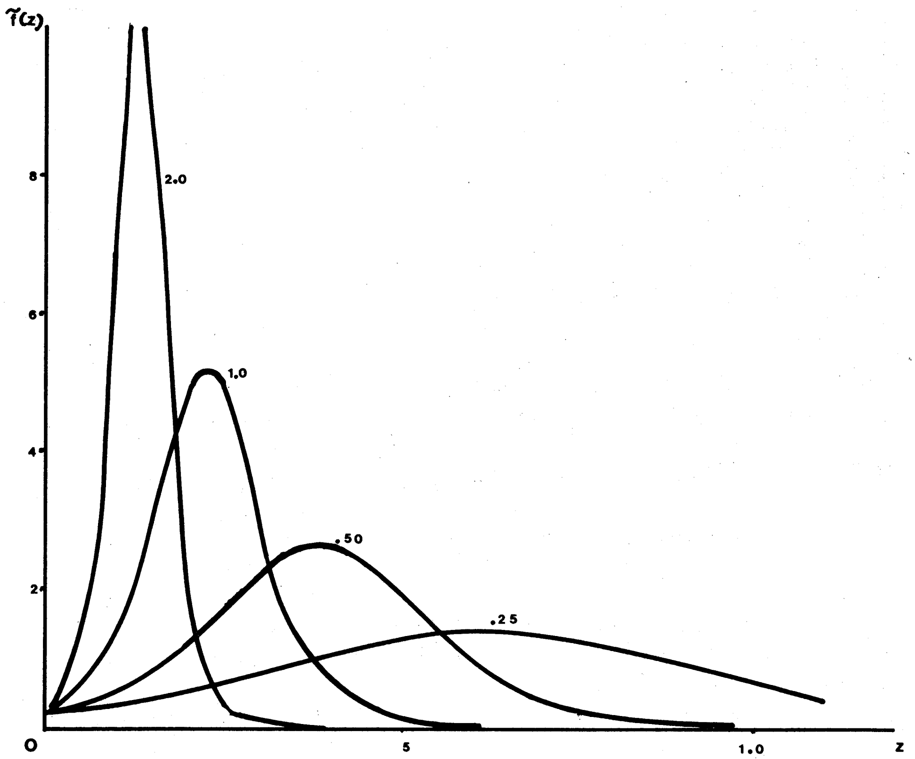

Fig. 10. The long-run average density functions under hypothesis (IN-i) for various values of μ (where $\lambda=0.05$ and $\nu=0.01$).

innovation is exactly what Schumpeter tried to capture by the word 'creative destruction'.

In order to avoid the possible confusion, let us emphasize once again that the long-run average frequency distribution of cost gaps, $\tilde{F}(z)$ or $\tilde{f}(z)$, is no more than a long-run statistical summary of the evolutionary pattern of the state of technology. It never implies that the industry's state of technology will, in the long run, converge to a static equilibrium. Far from it, the state of technology is a state of constant flux. As was vividly pictured in figs. 4 and 5, it is continuously moved by the force of imitation and discontinuously disrupted by the force of innovation. Its year-to-year or decade-to-decade evolution exhibits no tendency towards equilibrium. All that is claimed here is merely that if the long history of the development of the industry's state of technology is patiently studied, it is possible to detect the existence of certain statistical regularities out of its seemingly irregular evolutionary pattern.

7. The state of technology in the long run (II)

In the second special case, we introduce the following hypothesis concerning the nature of innovative activities.

Hypothesis (IN-b). Innovation is always carried out by a firm technologically most advanced at the time of innovation. Among those firms which are potentially capable of carrying out innovation the chance for success is equal at every point in time.

This hypothesis is, of course, an opposite extreme of hypothesis (IN-a) which insisted that every firm, whether technologically advanced or not, is potentially capable of striking innovation. Needless to say, it corresponds to the special case we examined in section 4. Although we found it easy to illustrate, by means of a diagram, the evolutionary pattern of the state of technology in this case, the analysis of its long-run average performance turns out to be slightly more involved.

Let $\xi \Delta t$ represent the probability that one of the technologically most advanced firms succeeds in carrying out an innovation during a small time interval Δt; where ξ is a very small positive constant. Then, the foregoing hypothesis can be restated more formally in the following manner. Suppose that the best practice production method at time t has unit cost equal to c_N which was introduced into an industry at time $T(c_N)(\leqq t)$. Then the number of firms which employ this production method at time t can be computed as $F_t(c_N) \cdot M$. Since the hypothesis (IN-b) insists that only those firms whose unit cost is c_N are potentially capable of introducing a new and better production method c_{N+1} and that any of those potential innovators has an equal chance for success, the probability that an innovation occurs during a small time interval between t and $t+\Delta t$ must be equal to $\xi \Delta t$ times the number of those firms given above, or

$$(\xi\Delta t)\cdot(F_t(c_N)M)=\frac{M\xi\Delta t}{1+(\mathrm{M}-1)\exp[-\mu(t-T(c_N))]}. \tag{15}$$

Consider the sequence of successive waiting times for innovation, $T(c_2)-T(c_1), T(c_3)-T(c_2), \ldots, T(c_N)-T(c_{N-1}), \ldots$. Under hypothesis (IN-b) they can be regarded as random variables which are identically distributed and independent of each other. In fact, that hypothesis enables us to compute explicitly the probability distribution of each waiting time. Let $U(s)$ denote the cumulative probability distribution of the waiting time $s \geqq T(c_N)-T(c_{N-1})$. Then a calculation whose detail is relegated to the appendix shows that it has the form of

$$U(s)=1-\left[\frac{M-1}{M}+\frac{1}{M}\exp(\mu s)\right]^{-\xi M/\mu} \tag{16}$$

for $s \geqq 0$. From this we can also calculate the expected waiting time for innovation τ as

$$\tau \equiv \mathrm{E}\{T(c_N)-T(c_{N-1})\}=\sum_{n=0}^{\infty}\left(\frac{M-1}{M}\right)^n \frac{1}{\mu n+\xi M}, \tag{17}$$

which is a decreasing function of ξ and μ. (See the appendix for the derivation.) The waiting time for innovation is thus expected to shorten as the effectiveness of innovative or imitative activity tends to increase.

In contrast to the first case, the probability of an innovation is uneven under hypothesis (IN-b). The probability of the next innovation is very small immediately after the occurrence of one innovation (for there is only one firm capable of striking it), but, as more and more firms succeed in imitating the best practice method and become potential innovators, this probability rises accordingly until almost the whole population of firms become capable of innovation. As time goes on, however, innovation takes place over and over again. After a sufficient number of years, therefore, a certain statistical regularity is expected to emerge out of the seemingly uneven pattern of the occurrence of innovations. In fact, the sequence of waiting times $\{T(c_{N+1})-T(c_N)\}$ constitutes what is in the probability theory called a 'renewal process', which is known to have well-behaved asymptotic properties. [See, for example, Feller (1966, ch. XI) for an excellent discussion of the theory of renewal process.] For instance, it is possible to show that the expected number of innovations per unit of time will in the long-run converge to a constant rate $1/\tau$, which is nothing but the reciprocal of the expected waiting time. (This is what is called the renewal theorem.)

The fact that the dynamic pattern of innovations will in the long run settle down to a statistical uniformity suggests to us that even under hypothesis (IN-b) the industry's state of technology as a whole will also exhibit a certain statistical tendency towards regularity. In order to show this, let us consider the behavior of $F_t^*(c)$, the expected cumulative frequency function at time t.

Under hypothesis (IN-b), we can indeed directly calculate $F_t^*(c)$ as follows. The cumulative capacity share $F_t(c)$ of a given unit cost c is zero before its innovation time $T(c)$ and remains so as long as no innovation occurs at and after $T(c)$. Its value jumps to $1/M$ at the time of the first innovation to occur at or after $T(c)$, and then follows a logistic growth path from that time on, independently of the pattern of innovations that follow. (Recall fig. 4.) Now, let $H_x(y)$ denote the probability that the length of time measured from a given time x to the first innovation to occur at or after x is equal to or less than $y(\geqq 0)$. We may call this the 'residual waiting time distribution'. In terms of this distribution, it is therefore easy to see that

$$F_t^*(c) = \int_0^{t-T(c)} \frac{1}{1+(M-1)\exp\{-\mu[t-(T(c)+y)]\}} \cdot \mathrm{d}H_{T(c)}(y) \tag{18}$$

for $t \geqq T(c)$.

The residual waiting time distribution $H_x(y)$ in general depends upon the calendar time x from which the residual waiting time y for the next innovation is measured. But, as time goes by, it will gradually get rid of this dependency and approach asymptotically to a steady-state distribution. More specifically, we have that

$$H_x(y) \to \int_0^y \frac{1-U(s)}{\tau} \cdot \mathrm{d}s \tag{19}$$

for a sufficiently large x. [See Feller (1966, p. 355) for the proof.] Accordingly, if we let both $T(c)$ and $t(\geqq T(c))$ grow large and substitute the explicit expression (16) for $U(s)$, we can in fact show that

$$F_t^*(c) \equiv \int_0^{t-T(c)} \{1+(M-1)\exp[-\mu(t-T(c)-y)]\}^{-1} \cdot \frac{1}{\tau} \cdot \left[\frac{M-1}{M} + \frac{1}{M}\exp(\mu y)\right]^{-(\xi M/\mu)} \cdot \mathrm{d}y \tag{20}$$

for $t \geqq T(c)$. Finally, if we note the relation: $z \equiv \ln c - \ln C(t) = \lambda(t-T(c))$ under hypothesis (PC), we can rewrite the asymptotic form of the expected cumulative capacity share function given above as follows:

$$F(z) \equiv \int_0^{z/\lambda} \left\{ 1+(M-1)\exp\left[-\mu\left(\frac{z}{\lambda}-y\right)\right]\right\}^{-1} \cdot \frac{1}{\tau}$$

$$\cdot \left[\frac{M-1}{M}+\frac{1}{M}\exp(\mu y)\right]^{-(\xi M/\mu)} \cdot \mathrm{d}y, \qquad (21)$$

which is independent of the calendar time t.

Fig. 11 illustrates a typical shape of $\tilde{F}(z)$.[10] As is seen from this diagram, the long-run average cumulative frequency function of cost gaps has a shape similar to the logistic growth curve even under hypothesis (IN-b). But, as is indicated by its density form $\tilde{f}(z) \equiv \mathrm{d}\tilde{F}(z)/\mathrm{d}z$ illustrated in fig. 12, it is, unlike the true logistic growth curve, skewed to the left. Figs. 13, 14 and 15 then illustrate numerically the influence of a change in each parameter value on the shape of the density form of the long-run average frequency function of cost gaps. The first diagram shows that an increase in the declining rate of

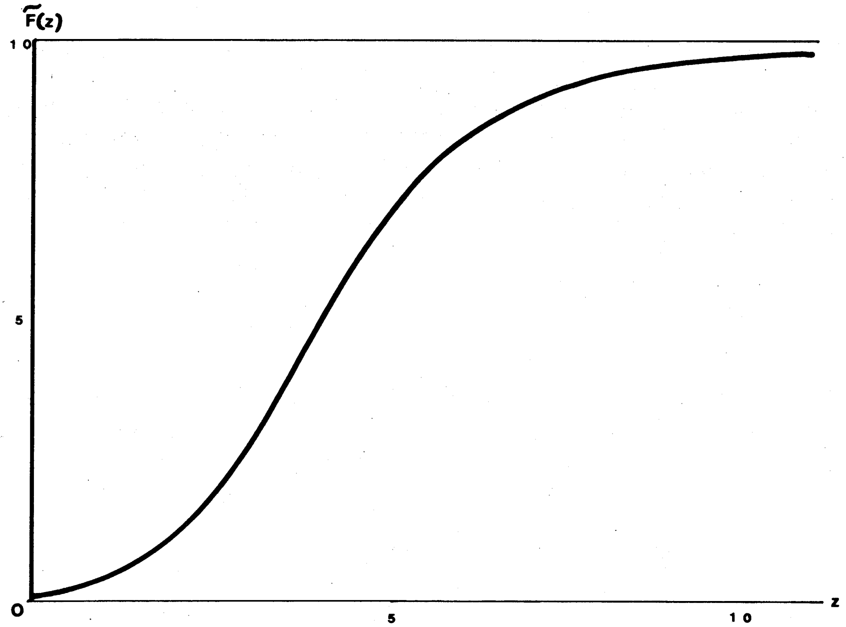

Fig. 11. The long-run average cumulative frequency function of cost gaps under hypothesis (IN-ii), when $\lambda=0.05$, $\xi=0.01$, $\mu=0.50$ and $M=20$).

[10]For this illustration, we have chosen the values of parameters as $\lambda=0.05$, $\xi=0.01$, $\mu=0.50$ and $M=20$.

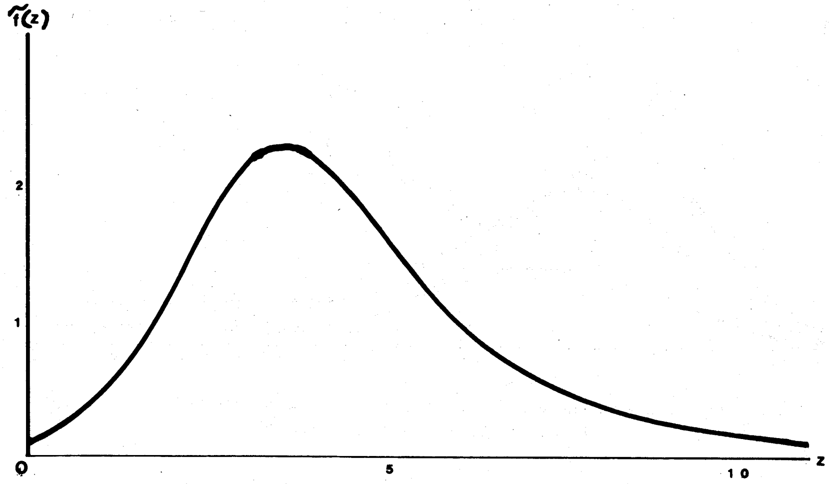

Fig. 12. The long-run average density function of cost gaps under hypothesis (IN-ii).

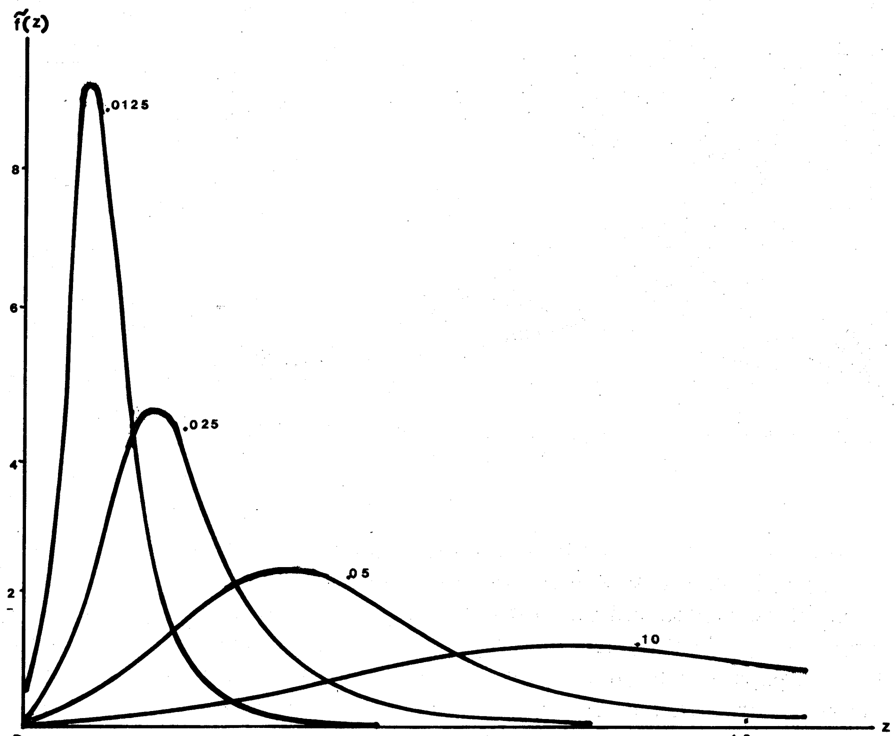

Fig. 13. The long-run average density functions under hypothesis (IN-ii) for various values of λ (where $\xi=0.01$, $\mu=0.50$ and $M=20$).

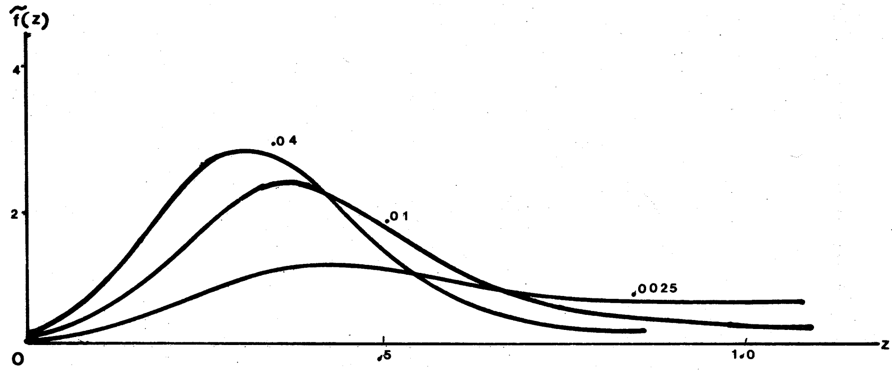

Fig. 14. The long-run average density functions under hypothesis (IN-ii) for various values of ξ (where $\lambda=0.05$, $\mu=0.50$ and $M=20$).

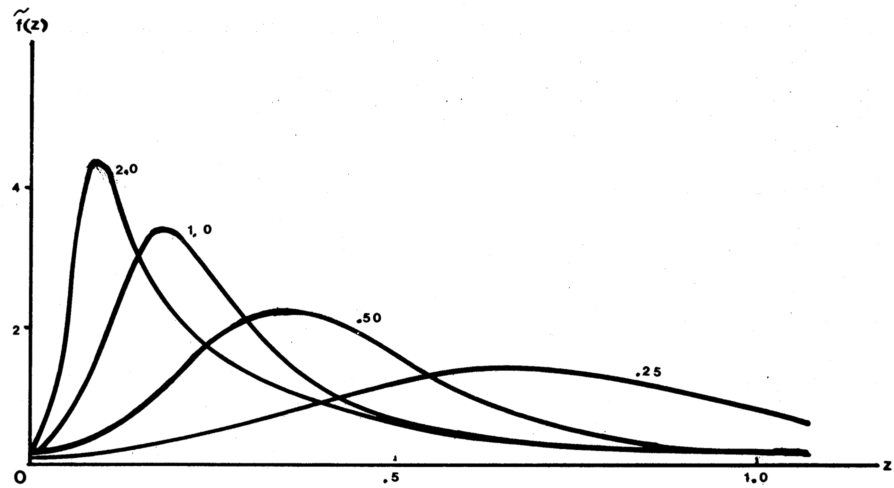

Fig. 15. The long-run average density functions under hypothesis (IN-ii) for various values of μ (where $\lambda=0.05$, $\xi=0.01$ and $M=20$).

potential unit cost λ tends to widen the average gap between unit costs and the potential unit cost and at the same time to disperse their distribution across firms. The second one shows that an increase in the rate of innovation (among the technological leaders) ξ has a tendency, albeit weak, to narrow the cost gaps and to make their distribution more concentrated. The third one shows that an increase in the rate of imitation μ also tends to narrow the cost gaps and concentrate their distribution. All these properties are similar to those of the long-run average density function of cost gaps under hypothesis (IN-a).

8. Turnover of firms

So far our analysis has supposed that there is no turnover of firms into and out of the industry and that the same M firms forever stay in it. It is, however, not so difficult to show that the models developed in this paper require no modification even if the process of turnover of firms is incorporated into them *as long as* (i) both entry and exit occur only within the class of firms employing the industry's worst practice production method and (ii) entry and exit just balance each other so that the total number of firms remains constant over time. Indeed, all the results we have obtained here are totally independent of the existence of firm-turnover under these assumptions.

In general, however, the process of turnover of firms does make a difference.

9. Concluding remarks

Our conclusion is our starting point. Let us recall the efficiency distribution of the metal stampings industry we presented in fig. 1. This paper began with a 'casual empiricism' that the state of technology in this industry (and in almost all industries in the U.S.) appears to be perpetually out of equilibrium, and then proceeded to demonstrate 'theoretically' that the state of technology will indeed be in perpetual disequilibrium under the Schumpeterian hypotheses on innovation and imitation. We have seen that while firms' imitation activities constitute an equilibrating force of technology which tends the industry's state of technology (not uniformly but logistically) towards a neoclassical equilibrium in which all the firms have full access to the most efficient production method available, the function of innovation lies precisely in upsetting this equilibrating tendency. It is the dynamic interaction between the continuous and equilibrating force of imitation and the discontinuous and disequilibrating force of innovation which governs the evolution of the industry's state of technology. In fact, we have been able to show how these two opposite forces will work hand in hand to generate a certain statistical regularity in the way in which the relative configuration of the distribution of efficiencies across firms develops itself over time. Under the joint pressure of imitation and innovation, the industry will not reach a neoclassical equilibrium with perfect technological knowledge even in the long run. While new technological knowledge constantly flows into the industry, actual production methods of a majority of firms always lag behind it, and a multitude of diverse production methods with a wide range of efficiencies will co-exist forever. Indeed, it is merely the statistical regularity of the relative pattern of these micro-scopic disequilibria that characterizes 'the long run' of the industry. We find it somewhat remarkable that the 'theoretical' shape of this statistical regularity we presented in fig. 7 or fig. 12 does resemble the 'empirical' shape of the efficiency distribution in fig. 1.

The only economic principle we have employed in the present paper is that of efficiency, namely, that firms always desire to adopt the more efficient or more profitable production method, whenever possible. (That they are not always able to do so is, of course, the basic premise of this paper.) All the results we have obtained here are therefore founded ultimately on this weakest of all economic principles. The task of the sequels is to introduce more specifically economic principles into the basic model and to work out their implications for our Schumpeterian dynamics. In particular, in the forthcoming paper (part II: Technological progress, firm growth and 'economic selection') we shall introduce another simple economic principle, that firms successful in innovation and imitation grow relatively faster than less successful ones, and study how the interplay of the process of firm growth and the process of technological innovation and imitation will mold the evolutionary pattern of the industry's state of technology.

Appendix

Let $U(s)$ be the cumulative probability of the waiting time for innovation $T(c_{N+1})-T(c_N)$. Suppose that none of the firms have been successful in innovation during a time interval $[0,s)$ after the last innovation time $T(c_N)$. Clearly, the probability of this occurrence is given by $1-U(s)$. On the other hand, by (15) the probability that one of the firms will introduce a new production method with c_{N+1} unit cost during the succeeding small time interval $[s,s+\Delta t]$ equals $\xi M\Delta t/\{1+(M-1)\exp[-\mu s]\}$. Since the probability that the production method with c_{N+1} unit cost will be introduced *for the first time* during the same small interval is the probability that no firms have been successful in $[0,s)$ *and* one of the firms becomes successful during $[s,s+\Delta t]$, we have the following equation:

$$U(s+\Delta t)-U(s)=(1-U(s))\cdot\{\xi M\Delta t/[1+(M-1)\exp(-\mu s)]\}. \tag{A.1}$$

By letting $\Delta t\to 0$, we obtain

$$\frac{\mathrm{d}U(s)}{\mathrm{d}t}=(1-U(s))\cdot[\xi M/\{1+(M-1)\exp(-\mu s)\}]. \tag{A.2}$$

This differential equation is not hard to solve, and we have

$$U(s)=1-\exp\left[-\xi M\int_0^s \frac{1}{1+(M-1)\exp(-\mu t)}\mathrm{d}t\right]$$

$$=1-\left[\frac{1}{M}\exp(\mu s)+\frac{M-1}{M}\right]^{-\xi M/\mu}. \qquad (A.3)$$

The expected waiting time is then calculated as

$$\tau\equiv \mathrm{E}\{T(c_{N+1})-T(c_N)\}=\int_0^\infty s\cdot \mathrm{d}U(s). \qquad (A.4)$$

By integrating the right-hand side by parts, we have

$$\int_0^\infty s\cdot \mathrm{d}U(s)=\int_0^\infty [1-U(s)]\,\mathrm{d}s=\int_0^\infty\left[\frac{M-1}{M}+\frac{1}{M}\exp(\mu s)\right]^{-\xi M/\mu}\mathrm{d}s$$

$$=\frac{1}{\mu}\cdot\int_0^1 t^{(\xi M/\mu)-1}\cdot\left(1-\frac{M-1}{M}t\right)^{-1}\cdot \mathrm{d}t$$

$$=\frac{1}{\mu}\cdot\sum_{n=0}^{\infty}\left[\left(\frac{M-1}{M}\right)^n\cdot\int_0^1 t^{n-1+(\xi M/\mu)}\,\mathrm{d}t\right]$$

$$=\sum_{n=0}^{\infty}\left(\frac{M-1}{M}\right)^n\cdot\frac{1}{\mu n+\xi M}.$$

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