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SCHUMPETERIAN DYNAMICS, PART II

Technological Progress, Firm Growth and 'Economic Selection'

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Business firms strive for survival and growth. They innovate in order to grow; they imitate in order to survive. The purpose of this paper, the second in the series on Schumpeterian dynamics, is to construct a simple mathematical model which is capable of studying the evolution of an industry's state of technology as a dynamic outcome of the interactions among innovation, imitation and capacity growth at the micro level of firms. The doctrine of 'economic selection' insists, by means of the analogy to the biological theory of natural selection, that only the most efficient firms will survive the long-run competitive struggle for capacity growth. Indeed, if neither innovation nor imitation is possible for the firm, a firm or a group of firms which is lucky enough to start with the most efficient technology will outgrow all the other firms in the long run. Once, however, the possibility of technological imitation is allowed for the firms, the industry will settle down to a static equilibrium of perfect technological knowledge, not by the success of the most efficient firms in their striving for the higher growth rate, but by the success of the less efficient in their effort to imitate the others. The blind force of economic selection is thus outwitted by the human force of imitative activities. Finally when firms are allowed to innovate in their technology, the selective force of market competition is no longer capable of weeding out the less fit even in the long run. Indeed, it is shown in the present paper that the dynamic interplay of the processes of innovation, imitation and growth will keep the industry's state of technology from settling down to the static equilibrium but reproduce in the long run a relative dispersion of efficiencies across firms in a statistically balanced form.

1. Introduction

Business firms strive for survival and growth. They innovate in order to grow; they imitate in order to survive. Firms which fail to innovate or imitate must go out of business or at least forego the opportunity to grow.

In the preceding paper (Part I: An evolutionary model of technological innovation and imitation), we studied how the dynamic processes of firms' innovation and imitation activities interact with each other and shape up the evolutionary pattern of the state of technology of an industry as a whole. We, however, did not take account of the differential impacts of such diverse technological developments among firms on their growth capabilities and the consequent repercussions on the evolutionary pattern of the state of technology itself. The first purpose of this paper, the second in the series on Schumpeterian dynamics, is to explore the evolution of the industry's state of

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technology as a dynamic outcome of the interactions between technological developments and growth processes at the micro level of firms.[1]

In his (too) well-known article on the methodology of positive economics, Milton Friedman (1953, p. 22) wrote:

> '...unless the behavior of businessmen in some way or other approximated behavior consistent with the maximization of returns, it seems unlikely that they would remain in business for long. Let the apparent immediate determinant of business behavior be anything at all — habitual reaction, random chance, or whatnot. Whenever this determinant happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not, the business will tend to lose resources and can be kept in existence only by the addition of resources from outside. The process of "natural selection" thus helps to validate the hypothesis — or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival.'

It is thus argued here that the force of competition, in particular, the force of dynamic competition for the acquisition of resources for growth is strong enough to ensure the survival and eventual dominance of the most efficient firms, whether their actions are guided by conscious maximization of returns or resulted from pure chance mechanism. It is the analogy between the biological competitive process in natural environment and the inter-firm competitive process in economic environment which has always been invoked in this kind of argument.[2] It is easy to claim metaphysically that only the fittest survives in the long run. It is, however, another matter to examine whether the logic of natural selection in biological evolutionary theory is indeed applicable to the description of the process of inter-firm competition for survival and growth in a genuinely economic environment. The second purpose of the present paper is to put the above 'economic selection' argument to scientific scrutiny, on the basis of our explicitly 'economic' model of evolutionary processes of firm growth and technological development.

2. State of technology

Let us begin by representing the 'state of technology' of an industry at a given point in time.

Suppose, as in the preceding paper, that there exist n distinct production

[1]See Nelson and Winter (1982) for related theoretical attempt in this direction.

[2]Besides Friedman (1953), Alchian (1950) developed a similar argument concerning the applicability of the doctrine of natural selection to the economic processes. See Winter (1975) for a perceptive criticism of this kind of argument.

methods coexisting in the industry at a given point in time. (The number of production methods, n, will of course change, i.e., increase over time as firms succeed in bringing new production methods into the industry.) Each production method is assumed to be of fixed proportion type, so that the unit cost is constant up to its productive capacity. If we denote by c the unit cost in terms of numeraire, all the existing production methods can be arrayed in accordance with their unit costs as follows:

$$c_n < c_{n-1} < \ldots < c_i < \ldots < c_1,$$

where c_n is the unit cost of the best practice method and c_1 that of the worst production method. Suppose, further, that the industry consists of M firms, including both active and potential producers. The cost conditions, of course, vary from firm to firm. Then, let $f_t(c_i)$ represent the relative frequency of firms with unit cost equal to c_i at time t, and let $F_t(c_i) \equiv \sum_{j=i}^{n} f_t(c_j)$ represent the relative frequency of firms with unit costs c_i *or less* at time t. We call $f_t(c)$ the frequency function and $F_t(c)$ the cumulative frequency function of unit costs at time t.

In the preceding paper we could represent the state of technology of an industry by $f_t(c)$ or, equivalently, by $F_t(c)$ alone. However, as soon as we take into consideration the dynamic interplays between firms' technological activities and their growth strategies, it is no longer sufficient to look at $f_t(c)$ or $F_t(c)$ alone in evaluating the industry's state of technology. It becomes necessary to study how the industry's total productive capacity is distributed over different production methods with different unit costs. Accordingly, let $k_t(c_i)$ represent the total volume of productive capacities with unit cost equal to c_i at time t, and let $K_t \equiv \sum_{j=1}^{n} k_t(c_j)$ be the total productive capacity of the industry as a whole at time t. [There is no logical difficulty in this calculation of total productive capacity within the context of our present model. See Sato (1975) for a useful discussion on the problem of capital aggregation.] We can then define by $k_t(c_i)/K_t$ the capacity share of unit cost c_i at time t. We denote this by $s_t(c_i)$ and call it the capacity share function. We then denote by $S_t(c_i) \equiv \sum_{j=i}^{n} s_t(c_j)$ the capacity share of unit costs from c_n to c_i at time t, and call it the cumulative capacity share function of unit costs. As a real life example, we present in fig. 1 capacity share functions and frequency functions of the metal stampings industry (SIC no. 3461) in 1958 and 1963. (The lower half of fig. 1 has reproduced fig. 1 of the preceding paper.) In this diagram, we have used the ratio of payrolls to value added as the proxy of unit cost and the share of value added as the proxy of capacity share. We have chosen the metal stampings industry among more than four hundred U.S. industries merely because it exhibits a fairly typical pattern of the state of technology.

Note in passing that the ratio between the capacity share function and the frequency function, i.e., $s_t(c_i)/f_t(c_i)$, represents the average capacity size of the

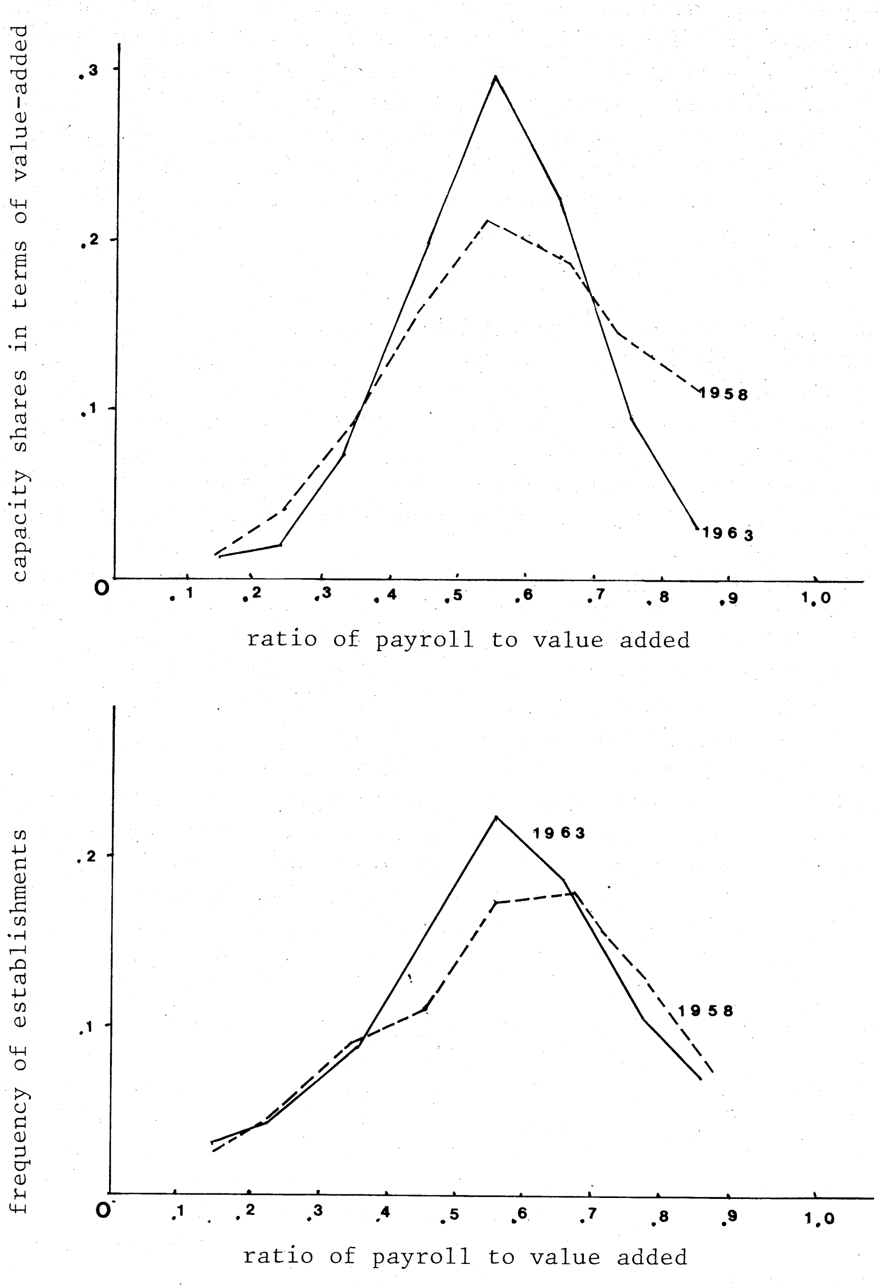

Fig. 1. The state of technology in metal stampings industry. *Source:* U.S. Department of Commerce (1968), 1963 Census of Manufactures, establishments classified by ratio of payrolls and value added. 1958 and 1963, MC 63(S)–8 (Government Printing Office, Washington, DC).

firms with unit cost c at time t (in terms of the average capacity size of all the firms in the industry).[3]

The state of technology of an industry as a whole at a given point in time is now represented by a pair of frequency function $f_t(c)$ and capacity share function $s_t(c)$ or, equivalently, by a pair of their cumulative counterparts, $F_t(c)$ and $S_t(c)$. The task of this paper then is to study how the state of technology, thus represented, evolves over time as a dynamic consequence of the complex interplay of innovation, imitation and growth processes of firms in an industry.

3. A model of firm growth

There are basically two causal mechanisms through which successful innovation or imitation leads to the growth of the firm. In the first place, a successful innovation or imitation and the consequent cost reduction allow the firm to lower the price of its product. In fact, the firm may choose to keep the profit margin constant and reduce the price proportionally to the cost reduction. This has the effect of lowering the price of its product relative to the less fortunate ones and directly promote the growth of its sales volume. Then, the growth of productive capacity follows suit. Alternatively, a successful innovation or imitation may allow the firm to increase its profit margin earned on each sales dollar and raise the rate of profit on the existing productive capacity. Such an increase in profit rate stimulates the firm's investment in productive capacity, either by influencing the expected profitability of investment or, to the extent that capital markets are imperfect, by directly providing an internal fund for investment projects.[4]

It is possible to translate these causal mechanisms into several simple mathematical structures. However, in order not to complicate the exposition, we shall discuss only the simplest model in the main text of this paper, leaving the discussion of alternatives to appendix A.

We shall consider an industry which consists of many firms producing an identical product. (This industry is exactly what we usually call a 'perfectly competitive' industry.) A firm in this industry is unable to control the price of its own product and takes the prevailing market price as given. It then follows that a firm successful in innovation or imitation necessarily experiences an increase in its profit margin, which works to stimulate its capital investment. The first causal mechanism from technological progress to capacity growth is thus ignored, and only the second one will be studied explicitly in what follows.[5]

[3]We shall discuss the long-run properties of the distribution of average capacity size in section 7.

[4]Scherer, for instance, found in his empirical study (1965) the prevalence of the first causal mechanism.

[5]The first example in appendix A deals with the first causal mechanism.

Let $\bar{p}(t)$ be this market price prevailing at time t. [The discussion of how this market price is determined at each point in time is postponed to the following paper (Part III); the following story has a structure that is independent of the way the market price is determined.] Then, we can calculate the 'profit margin' of the firm whose unit cost equals c at time t as $(\bar{p}(t)-c)/\bar{p}(t)$, which we approximate as $\ln \bar{p}(t) - \ln c$. (This is not a bad approximation, as long as $\bar{p}(t)$ and c are not so wide apart.) If we suppose that the value ratio between output and capacity (or capital stock) varies little from firm to firm as well as from time to time, this index of profit margin can be regarded as a good proxy for the rate of profit for the firm. (This is of course true only for the firm operating at full capacity. The rate of profit of the firm operating below its capacity has to be adjusted by the factor equal to its capacity utilization rate, and that of a firm which has stopped producing is obviously zero.) There must therefore exist a positive correlation between the rate of capacity growth of a firm and its current profit margin, either through the latter's influence on the expected future profitability or as the latter's being the internal fund for capital investment. As a first-order approximation, we set up the following hypothesis:

Hypothesis (G). The rate of capacity growth of a firm whose unit cost is c at time t is (approximately) equal to

$$\gamma \cdot (\ln \bar{p}(t) - \ln c) + \gamma_0, \tag{1}$$

where $\gamma > 0$ is a positive constant representing the effect of the profit margin and γ_0 is a possibly negative constant which summarizes all the factors other than profit margin, such as rates of interest, user's costs, animal spirit, etc., which influence the firm's investment policy.

Studies on the process of firm growth have in the past centered around the so-called 'Gibrat's law of proportionate effect' [Gibrat (1931)]. In its weakest form, this is presented as an empirical law asserting that the expected growth rate of a firm during a specific period is independent of the initial size of the firm. It, in fact, claims that the current size of a firm is no more than a cumulative result of the past growths and in itself has little causative role to play in predicting the current growth rate of the firm. Hypothesis (G) given above is compatible with this weakest form of Gibrat's law, for it does not have the firm size as the explanatory variable of the growth rate. However, Gibrat's law is usually formulated in a stronger form. In its strong form it insists that the probability that a firm grows at a given rate is constant over time and uniform across firms. Needless to say, our hypothesis (G) does violate this strong version of Gibrat's law. A firm, once successful in innovation or imitation, is able to enjoy a higher growth rate at least for some duration of time. In fact, hypothesis (G) implies that it is this sustenance of higher growth rate that is the reward of the success in innovation or imitation. Empirical studies on firm growth processes have almost invariably rejected the strong version of Gibrat's law, although they are often supportive of its weakest form.[6]

Now, most of the theoretical works on the size distribution of firms rely on the strong version of Gibrat's law. Starting from the premise that growth rates of firms are governed completely by pure chance mechanism, they deduce, as the long-run average size distribution of firms, a long-tailed, log-normal or Yule distribution, which fits empirical distributions quite well.[7] Relatively recent works of Ijiri and Simon attempt to extend these results by incorporating serial correlation of growth rates, but they still leave the stochastic mechanism governing the growth process of firms totally in the black box.[8] Although the present paper is not concerned with explaining the size distribution of firms, one of the objectives of the following investigation is to connect the firm growth process to the outcomes of the competitive struggles among firms for technological supremacy in an industry. It thus can be regarded as an attempt at the endogenization of Gibrat's law.

4. The logic of economic selection

We are now in a position to embark upon the detailed analysis of the evolutionary process of an industry's state of technology, under the combined pressure of firms' capacity growth, technological innovation and imitation. In order to put into relief the effect of the differential growth rates among firms with diverse cost conditions, let us begin our analysis by ignoring the effects of technological innovation and imitation. They will be introduced into the analysis in the sections that will follow.

Now, under the supposition of no technological innovation and imitation, the frequency function $f_t(c)$, and the cumulative frequency function, $F_t(c)$, of unit costs are both invariant over time. Efficient firms are forever efficient, and inefficient ones forever inefficient. The capacity share function $s_t(c)$ and the cumulative capacity share function $S_t(c)$ do, however, change over time, in reponse to different growth rates among firms. Their evolutionary patterns must be studied in detail.

For this purpose, differentiate the definition of the capacity share function, $s_t(c_i) \equiv k_t(c_i)/K_t$, with respect to time, and we obtain:

$$\frac{\dot{s}_t(c_i)}{s_t(c_i)} = \frac{\dot{k}_t(c_i)}{k_t(c_i)} - \frac{\dot{K}_t}{K_t} = \frac{\dot{k}_t(c_i)}{k_t(c_i)} - \sum_{j=1}^{n} \frac{\dot{k}_t(c_j)}{k_t(c_j)} \cdot s_t(c_j), \tag{2}$$

[6]See, for example, Mansfield (1962), Steindl (1965), and Singh and Wittingenton (1968, 1975).
[7]See Hart and Prais (1956), Adelman (1958), and Ijiri and Simon (1977, chs. 1 and 7).
[8]Ijiri and Simon (1977, chs. 8, 9 and 11).

where $\dot{x} \equiv dx/dt$. Since, in virtue of hypothesis (G) firms with the same unit cost grow at the same rate irrespective of their capacity sizes, we can substitute the expression (1) into $\dot{k}_t(c_i)/k_t(c_i)$ in eq. (2) and obtain

$$\frac{\dot{s}_t(c_i)}{s_t(c_i)} = -\gamma \cdot (\ln c_i - \ln \bar{c}(t)), \tag{3}$$

where $\bar{c}(t)$ represents the industry-wide average unit cost at time t, defined by

$$\ln \bar{c}(t) \equiv \sum_{i=1}^{n} s_t(c_i) \cdot \ln c_i. \tag{4}$$

[This should not be confused with the notion of the potential unit cost $C(t)$.] The growth rate of the capacity share of a given unit cost, $\dot{s}_t(c_i)/s_t(c_i)$, is thus shown to be proportional to the extent of its relative advantage over the industry average unit cost, $-(\ln c_i - \ln \bar{c}(t))$. In other words, the lower the unit cost relative to that industry average, the more rapidly will the productive capacity with the unit cost gain its share; and the higher the unit cost relative to the industry average, the speedier will the productive capacity with that unit cost lose its share in the industry.

Now eq. (4) says that the share of productive capacities which have a 'cost advantage' in the sense of $c_i < \bar{c}(t)$ grows at a rate which is proportional to the relative size of that cost advantage. It thus appears to grow exponentially at this rate over time and hence exceed any positive number eventually. But, of course, the capacity share can never exceed unity by being a relative fraction! Something was wrong with this reasoning, and it is not difficult to locate where it went wrong. Indeed, it is only necessary to recall the definition (4) of the average unit cost $\bar{c}(t)$. It is plain from this that the average unit cost is not a given constant, but a weighted geometrical average of existing unit costs with weights being their corresponding capacity shares, which grow or contract according to the very dynamic eq. (3) we are analyzing. As time goes by, productive capacities with lower-than-average unit costs grow relatively more than those with higher-than-average unit costs. This has an effect of shifting weights in favor of lower-than-average unit costs, thereby reducing the weighted averaged unit cost $\bar{c}(t)$. Such a decline of the average unit cost undercuts the existing unit costs one by one and has these capacity shares contract until the productive capacities with the least unit cost dominate the entire industry and hence the value of the average unit cost is reduced to the level of this least unit cost.

In order to formalize the above argument more rigorously, let us differentiate the definition (4) of the average unit cost logarithmically with respect to time. We then have

$$\frac{\mathrm{d}\ln \bar{c}(t)}{\mathrm{d}t}=\frac{\dot{\bar{c}}(t)}{\bar{c}(t)}=\sum_{i=1}^{n} \ln c_i \cdot \dot{s}_t(c_i).$$

Substituting eq. (3) for the growth rate of capacity share and re-arranging terms, we obtain that

$$\frac{\dot{\bar{c}}(t)}{\bar{c}(t)}=-\gamma\cdot\sum_{i=1}^{n}(\ln c_i-\ln \bar{c}(t))^2\cdot s_t(c_i). \tag{5}$$

Hence, the rate of *decrease* in the average unit cost, $-\dot{\bar{c}}(t)/\bar{c}(t)$, is proportional to the 'weighted variance' of the logarithmic values of unit costs. The average unit cost therefore keeps declining as long as the shares of the firms which do not have the least cost technique are non-negligible. It stops declining only when the share of the least cost firms approaches unity and the cross-section variance of unit costs becomes zero. The only possible long-run equilibrium in this model without technological progress is the situation in which the least cost firms dominate the entire industry, by outgrowing all the other firms. Only the most efficient survive in the long run. This is precisely the world of 'the survival of the fittest', in which the logic of 'economic selection' has the complete upperhand. Indeed, eq. (5) is formally equivalent to what biological evolutionary theorists and population geneticists call the 'fundamental equation of natural selection'. This equation was first derived by R. A. Fisher and has since been playing a fundamental role in the theory of natural selection in biology.[9] We may, accordingly, call our eq. (5) the 'fundamental equation of economic selection'. In the economic world with neither innovation nor imitation the logic of economic selection is exactly that of natural selection.

In the following sections, however, we shall examine whether the doctrine of economic selection itself is able to survive once the processes of imitation and innovation are explicitly introduced into our evolutionary model of the industry. As a preparation for this task, we find it convenient to reformulate the logic of economic selection from a slightly different angle.

This time, let us differentiate the cumulative capacity share function with respect to time. We then obtain

$$\dot{S}_t(c_i)\equiv\sum_{j=i}^{n}\dot{s}_t(c_j)=\sum_{j=i}^{n}\{-\gamma(\ln c_j-\ln \bar{c}(t))\cdot s_t(c_j)\}$$

$$=[\gamma\cdot\delta_t(c_i)]\cdot S_t(c_i)\cdot[1-S_t(c_i)], \tag{6}$$

[9]Fisher (1958). See, for example, Crow and Kimura (1970) for the more recent treatment of the fundamental equation of the natural selection.

where $\delta_t(c_i) > 0$ is defined by

$$\delta_t(c_i) \equiv \sum_{j=1}^{i-1} \ln c_j \cdot s_t(c_j) \Big/ \sum_{j=1}^{i-1} s_t(c_j) - \sum_{j=i}^{n} \ln c_j \cdot s_t(c_j) \Big/ \sum_{j=i}^{n} s_t(c_j), \tag{7}$$

for $i = 2, 3, \ldots, n$. The function $\delta_t(c_i)$ thus defined represents the difference between the logarithmic average of the subset of unit costs which are at least as high as c_{i-1} and the logarithmic average among those which are at least as low as c_i. If the capacity share function has the form of a uniform distribution, $s_t(c_i) = 1/n$ where $\ln c_i$ is located at $\alpha - \beta i$ for $i = 1, 2, \ldots, n$, then $\delta_t(c_i) = \beta n/2$ for all i and t. Or, if the number of distinct unit costs is two, $\delta_t(c_2) = \ln c_1 - \ln c_2$. Although $\delta_t(c_i)$ cannot be treated as a given constant in general, we can still expect it to fluctuate little from one unit cost to another and from one point in time to another. In fact, from now on, we proceed our analysis as if the value of $\delta_t(c_i)$ were a given positive constant, and substitute for it a constant parameter δ (>0). It should be borne in mind, however, that this approximation is solely for expositional convenience. We have already completed our discussion of the logic of economic selection once. In any case, this approximation allows us to rewrite eq. (6) as

$$\dot{S}_t(c_i) = (\gamma\delta) \cdot S_t(c_i) \cdot [1 - S_t(c_i)] \quad \text{for} \quad i = n, n-1, \ldots, 1, \tag{8}$$

which can be solved explicitly to yield

$$S_t(c_i) = 1/\{1 + (1/S_T(c_i) - 1) \cdot \exp[-(\gamma\delta) \cdot (t - T)]\}, \tag{9}$$

where T ($\leqq t$) is a given initial time.

We have again encountered a 'logistic growth equation' we have so much familiarized ourselves with in Part I of the present series of the papers. This time, however, its derivation was based on an entirely different principle.

The upper half of fig. 2 illustrates how the force of economic selection, working through differential growth rates between low cost and high cost firms, sets in motion the family of cumulative capacity shares along logistic growth paths. For instance, the capacity share of the least unit cost, $S_t(c_n) = s_t(c_n)$, can grow almost exponentially when it is small. But, as it begins to occupy a non-negligible portion and the weights of the industry average unit cost $\bar{c}(t)$ begin to shift towards the lower unit costs, the average unit cost $\bar{c}(t)$ begins to decline as well, and the relative cost advantage of the least cost firms gradually disappears. The capacity share of the least cost firms therefore lags behind the exponential growth path of the initial stage, and decelerates its growth momentum as it becomes larger and larger. It never stops growing, however. With decelerating speed, it nonetheless approaches unity asymptotically. The capacity shares (but not the cumulative capacity shares) of the less efficient production methods, $s_t(c_{n-1}), s_t(c_{n-2}), \ldots, s_t(c_1)$, on

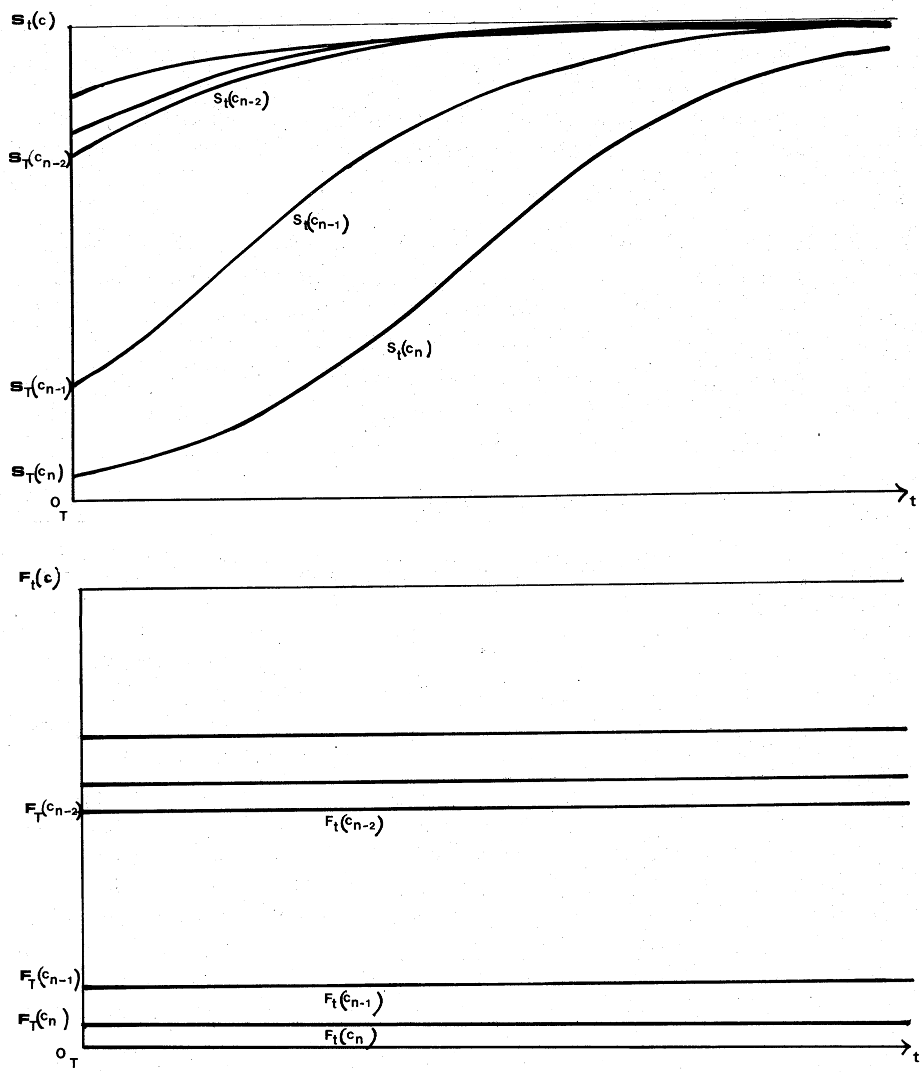

Fig. 2 The evolution of the state of technology under the sole pressure of economic selection.

the other hand, dwindle gradually over time (though some of them may grow temporarily before they start dwindling) and disappear entirely in the long run. As a comparison, the lower half of fig. 2 exhibits the movement of the family of cumulative frequencies of firms. In the industry with neither innovation nor imitation, they remain constant forever.

5. Imitation and economic Lamarckism

We shall now combine the model of economic selection we have so far developed with the model of technological imitation and innovation we presented in Part I of this series of papers. In the first place, let us recall the following hypothesis which captures the so-called snow-ball effect or band-wagon effect, characterizing the diffusion process of technology through firms' imitation activities.

Hypothesis (IM). The probability that a firm is able to copy a particular production method is proportional to the share of total productive capacity which employs that method in the period in question. The firm, of course, implements only the method whose unit cost is lower than the one it is currently using. Formally, this hypothesis says that the probability that a firm with unit cost c_i imitates a production method of unit cost c during a small time interval $[t, t+\Delta t]$ is equal to

$$\mu s_t(c)\Delta t \text{ for } c<c_i \quad \text{and} \quad 0 \text{ for } c \geqq c_i, \tag{10}$$

where $\mu>0$ is a parameter summarizing the effectiveness of the firm's imitation activity.

Hypothesis (IM′) we introduced in the preceding paper supposed that the greater the number of firms employing a given production method the easier for another firm to imitate that method, regardless of the size of those firms. The new hypothesis (IM) proposed above has modified this hypothesis by insisting that not only the number of the firms employing a production method but also the size of each of these firms affects the easiness of imitating that method. The secret of production method is more likely to leak outside, as the more and more firms employ it and the larger and larger they become. On the other hand, its own size does not provide the imitator with any particular advantage in the probability of imitation, although, once a better production method is successfully copied, the assumed disembodied nature of technological change confers the proportionally larger fruits on the larger size firm.

Under the modified hypothesis (IM), it is not difficult to apply the argument similar to the one given in section 2 of the preceding paper and obtain the following differential equation which describes how the firms' imitation activities move the industry's cumulative share function:

$$\dot{S}_t(c_i) = \mu \cdot S_t(c_i) \cdot [1 - S_t(c_i)]. \tag{11}$$

To see this, let us first note that the value of $S_t(c_i)$ changes whenever one of the firms whose unit costs are higher than c_i succeeds in imitating one of the production methods with unit cost c_i or less. In fact, it increases by the magnitude equal to the capacity share of the imitator. [The value of $S_t(c_i)$ remains unperturbed so long as the pre-imitation unit cost of an imitator was c_i or less, for only an infra-marginal exchange of capacity shares is effected in such a case.] Now, according to hypothesis (IM), the probability that one of the firms with unit costs higher than c_i can imitate one of the production methods of unit costs c_i or less is equal to $\mu\cdot\{s_t(c_i)+\cdots+s_t(c_n)\}\cdot\Delta t=\mu\cdot S_t(c_i)\cdot\Delta t$ during a time interval $[t,t+\Delta t]$. Hence, the *expected* increase in the value of $S_t(c_i)$, due to a successful imitation of *this* particular firm, during the same time interval, is equal to this probability times the firm's capacity share. Since the total capacity share of the firms with unit costs higher than c_i is by definition given by $1-S_t(c_i)$, the *expected* increase in the value of $S_t(c_i)$ to be brought about by a successful imitation of *one of* those firms can be computed as $(\mu S_t(c_i)\Delta t)\cdot(1-S_t(c_i))$. The so-called law of large numbers then allows us to use this as a good approximation of the *actual* increase in the value of $S_t(c_i)$. If we divide this expression by Δt and let Δt approach zero, we finally obtain eq. (11). This equation, however, has not taken account of the effect of the differential growth rates among firms with different cost conditions, which was described by eq. (8) of the preceding section. If, therefore, we add (8) and (11), we obtain a new logistic differential equation which describes the combined effect of the processes of capacity growth and technological imitation,[10]

$$\dot{S}_t(c_i)=(\mu+\gamma\delta)\cdot S_t(c_i)\cdot[1-S_t(c_i)]. \tag{12}$$

Solving this explicitly, we obtain a new logistic growth path of the form

$$S_t(c_i)=1/\{1+(1/S_T(c_i)-1)\cdot\exp[(\mu+\gamma\delta)(t-T)]\}, \tag{13}$$

for $t\geqq T$. Under the combined pressure of capacity growth and technological imitation, the cumulative capacity share function $S_t(c)$ will thus follow a familiar logistic growth path, illustrated by the upper half of fig. 3. The only formal difference from the preceding case of no technological imitation is that its growth parameter is now the sum of $\gamma\delta$ and μ — the sum of the parameter representing the effect of differential growth rates and the parameter representing the effect of the imitation process. In the long run, therefore, the least cost production method will again completely dominate the industry's total productive capacity. [That is, $S_t(c_n)\to 1$ as $t\to\infty$.]

The process of capacity growth and the process of technological imitation, however, contribute to the logistic growth process of the cumulative capacity share function for entirely opposite reasons. While, as was shown in the preceding section, the former represents the force which tends to amass the

[10]It is easy to show that we can indeed *add* these two effects as long as time is continuous.

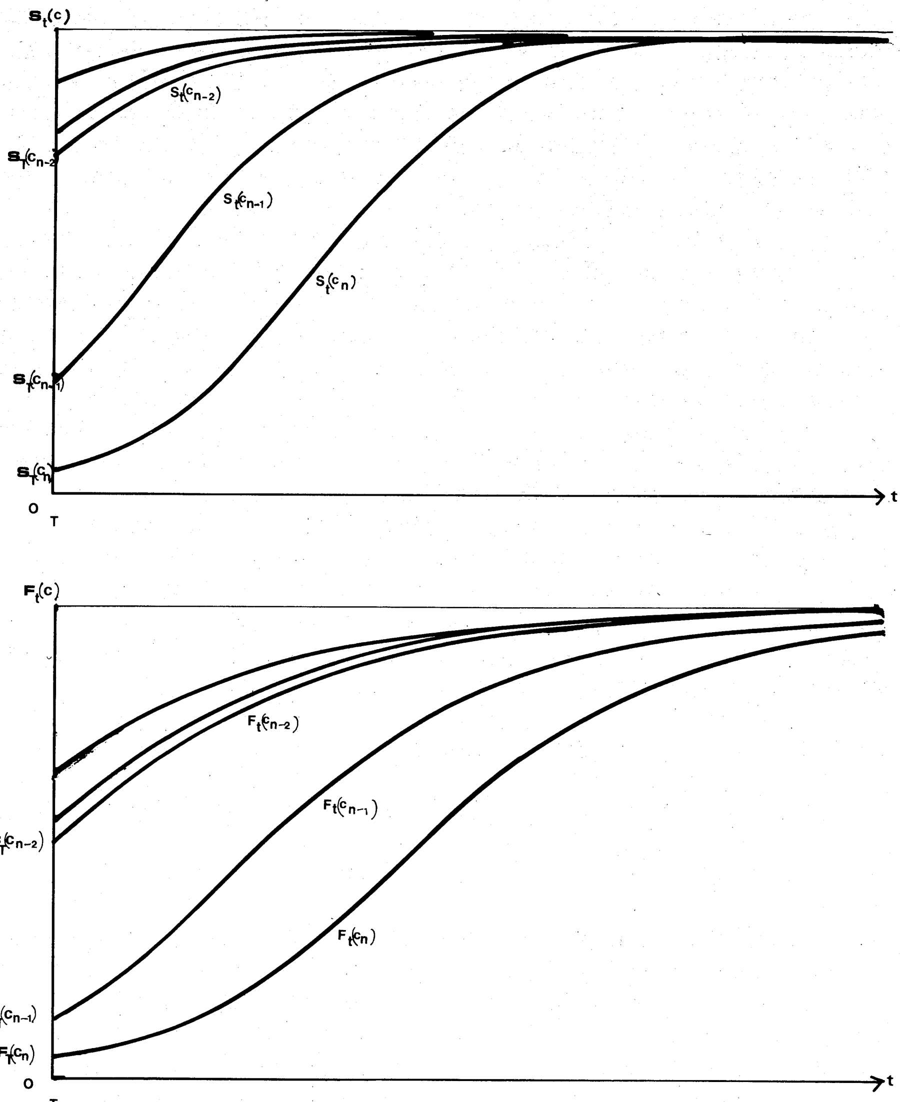

Fig. 3. The evolution of the state of technology under the combined pressure of economic selection and technological imitation.

industry's productive capacities in the hands of few technologically advanced firms through their maximal capability of capacity growth, the latter represents the force which dissipates the advantage of the low cost production methods among all firms through their imitation efforts. While the former represents a centralizing tendency, the latter represents a decentralizing tendency of productive capacities.

In order to see in more detail how these two opposite tendencies will interact with each other, let us now turn our attention to the evolutionary pattern of the cumulative frequency function of unit costs, $F_t(c)$. Indeed, a slight modification of the argument employed in deducing the logistic differential equation (11) leads us to the following differential equation:

$$\dot{F}_t(c_i) = \mu \cdot S_t(c_i) \cdot (1 - F_t(c_i)). \tag{14}$$

Here, $(1 - F_t(c_i))$ in the right-hand side represents the fraction of the firms whose unit costs are higher than c_i, and $\mu \cdot S_t(c_i)$ the probability per unit of time that one of these firms succeeds in imitating one of the production methods with unit cost c_i or less. The expected rate of increase in the value of $F_t(c_i)$, therefore, equals their product, so that an application of the strong law of large numbers gives us differential equation (14). As is shown in appendix B, if eq. (14) is paired with (13), it is possible to solve it to deduce the following explicit formula for the growth path of $F_t(c)$:

$$F_t(c_i) = 1 - (1 - F_T(c_i))\{(1 - S_T(c_i)) + S_T(c_i) \cdot \exp[(\gamma\delta + \mu)(t - T)]\}^{-\mu/(\gamma\delta + \mu)} \tag{15}$$

for $t \geqq T$ and for all $i = 1, 2, \ldots, n$. As is illustrated in the lower half of fig. 3, the frequency of the firms employing the best practice method $F_t(c_n)$, its lowest layer, grows slowly at first, accelerates its speed as the corresponding capacity increases its share, but after the corresponding capacity share reaches its midpoint, loses its growth momentum, yet approaching unity asymptotically. [That is, $F_t(c_n) \to 1$, as $t \to \infty$.] In the long run, therefore, all the firms in the industry will come to adopt the best practice method with unit cost c_n.

In the economy with no technological imitation (nor innovation), the firms which are lucky enough to possess the least cost production method and hence able to afford the highest growth rate will in the long run monopolize the whole productive capacity of the industry. However, as soon as the possibility of technological imitation by relatively high cost firms is taken into account, this logic of economic selection loses much of its effectiveness. True that the lowest cost firms will again monopolize the whole productive capacity in the long run, but, as we have seen above, the force of technological imitation will eventually allow *all* the existing firms to join the rank of the lowest cost firms. In fact, it is precisely the very expansion of the productive capacity of the lowest cost firms — their own success — which necessarily invites the vigorous imitation activities of the less fortunate ones and betrays their own bids for the dominance of the whole industry. The human force of imitation thus has the power to overcome the blind force of economic selection. It is, in other words, the 'Lamarckian' mechanism, not the

Darwinean, that assures the survival of the firms in this world of capacity growth and technological imitation.

Still only the fittest survives in this world, and the industry's long-run state of technology is nothing but a neoclassical paradigm of perfect information. The introduction of technical innovation, however, will destroy this last vestige, as we shall see in the sections that follow.

6. Innovation and the state of technology in the long run

Finally, let us introduce the process of technological innovation into our picture of the industry. To begin with, we follow the notation of the previous paper and denote by $T(c)$ the 'innovation time' of a unit cost c, i.e., the time at which a production method of unit cost c is for the first time put into practice in the industry. Then, at each innovation time $T(c)$, both the cumulative capacity and the cumulative frequency of the unit cost c, $S_t(c)$ and $F_t(c)$, emerge out of nothingness and start their long evolutionary journey. Since the total number of firms in the industry is M, the initial firm-frequency $F_{T(c)}(c)$ is equal to $1/M$. On the other hand, the initial capacity share $S_{T(c)}(c)$ is not necessarily equal to $1/M$. It may be larger or smaller than $1/M$, depending upon whether the capacity size of the innovator is above or below the industry average. The upper half of fig. 4 illustrates the evolutionary pattern of the cumulative capacity share function and the lower half the evolutionary pattern of the cumulative frequency function, both under the joint pressure of differential capacity growths, technological imitation and innovation. They are drawn under the assumption that every firm is able to innovate.

The state of technology is moved by the interplay of three kinds of dynamic forces. In the first place, the force of economic selection works to make the logistic growth of cumulative capacity shares, shown in the upper half of fig. 4, speedier than the growth of cumulative frequencies, shown in the lower half of fig. 4. This is the force which tends to amass the industry's productive capacity in the hands of the lower cost firms. Second, technological imitation is the main motive force behind the logistic growth process of cumulative frequencies. (Of course, it also contributes to the logistic growth process of cumulative capacity shares.) This is the force which diffuses the efficient technology throughout the industry. Finally, we have technological innovation, whose function in the Schumpeterian world is to disrupt the tendency of the industry to settle down to neoclassical equilibrium and to force it to forever search for the higher and higher efficiency. Then, how will these conflicting forces interact with each other and fashion the state of technology in the long run?

In order to give an answer to this question, we need to restate the hypotheses, posited in Part I, which pertain to the process of inventive activities outside of the industry and the process of innovative activities

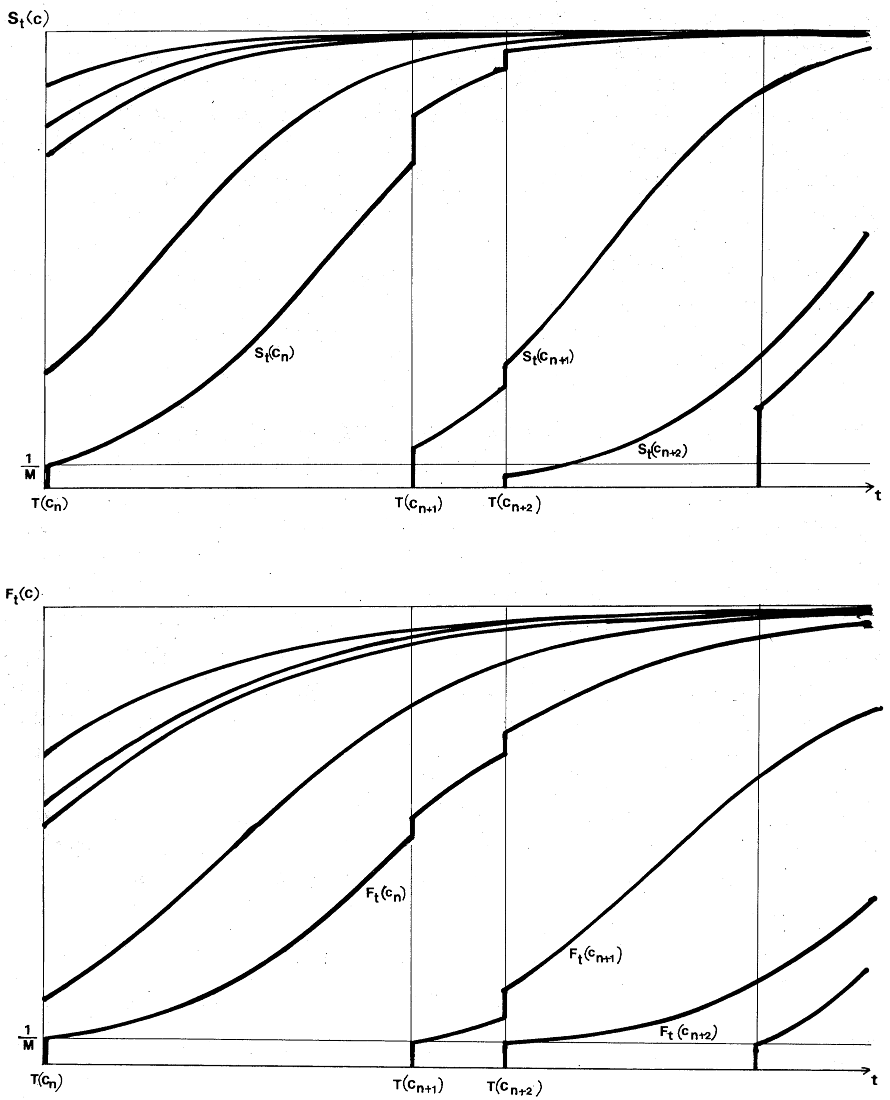

Fig. 4. The evolution of the state of technology under the combined pressure of economic selection, technological imitation and technological innovation.

within the industry. First, let $C(t)$ represent the 'potential unit cost' at time t, i.e., the unit cost of the best productive method technologically possible at time t. Only the firm successful in an innovation at time t can turn this technological possibility into practical use. The potential unit cost is supposed to be determined by the stock of scientific knowledge accumulated by basic research activities throughout the entire economy. Now, let $T(c)$ denote the inverse function of $C(t)$, defined by $T(C(t)) \equiv t$. We know that, if there occurs an innovation at time t, the unit cost of the innovator becomes equal to the potential unit cost $C(t)$. Hence, if a given production method with unit cost c is currently in use, it must have been introduced into the industry at time $T(c)$. This, of course, conforms to the notation $T(c)$ we have already adopted to denote the innovation time of a unit cost c. As in Part I, we assume

Hypothesis (PC). The potential unit cost is declining at a constant (positive) rate λ over time; that is

$$C(t) = \exp(-\lambda t). \tag{16}$$

Then, we can specify the innovation time as $T(c) = -(1/\lambda) \cdot \ln c$.[11]

Next, we have to introduce a hypothesis characterizing the condition for a firm to become an innovator. In Part I we considered two alternative hypotheses pertaining to this — one supposing every firm is able to innovate, the other assuming only the most efficient are capable of innovation. However, once effects of capacity growth are taken account of, the analysis of the long-run evolutionary pattern of the state of technology under the second alternative becomes quite intractible, though perhaps not an absolute impossibility.[12] In the present paper at least, we therefore have to content ourselves with the analysis of the long-run implications of the first alternative only. We thus posit here

Hypothesis (IN-a). Every firm has an equal and constant chance for successful innovation at every point in time. Specifically, the probability that a firm succeeds in carrying out an innovation during a small time interval Δt is equal to

$$v \cdot \Delta t, \tag{17}$$

where $v > 0$ is a constant parameter summarizing the effectiveness of the firm's innovation activity.

This hypothesis is identical with the one we posited in section 5 of Part I. Within the context of the present paper which takes an explicit account of the process of capacity growth, it can be interpreted as supposing implicitly that the size *per se* does not provide the firm with any advantage in the probability of its innovative success. At the same time, however, because of

[11] We have set $C(0) = 1$ for convenience.

[12] It is of course possible to examine this case by means of a computer simulation.

the assumed disembodied nature of technological change, the fruit of a successful innovation can be enjoyed by the firm in proportion to its existing capacity size. All in all, a kind of constant returns to scale is implicitly posited in respect to the firm's innovation activity in the above hypothesis.

Now, according to hypothesis (IN-a), the probability that *one of M* firms in the industry suceeds in an innovation during a small time interval is equal to $(\nu\Delta t)\cdot M=(\nu M)\cdot\Delta t$. Hence, the occurrence of innovations in the industry as a whole is subject to a Poisson process, which is sometimes called the law of rare events. As time goes on, however, the industry undergoes many innovations. And out of such repeated occurrence of rare events a certain statistical regularity is expected to emerge. Indeed, not only the pattern of innovations but also the whole evolutionary pattern of the state of technology is expected to exhibit a certain statistical regularity in the long run.

To see this, let $S_t^*(c)$ and $F_t^*(c)$ denote, respectively, the expected value of the cumulative capacity share function and the expected value of the cumulative frequency function at time t. For the purpose of describing the long-run average pattern of the industry's state of technology, we only have to concern ourselves with the behavior of $S_t^*(c)$ and $F_t^*(c)$.

Under hypotheses (G), (IM), (PC) and (IN-a), we are able to derive the following pair of differential equations concerning $S_t^*(c)$ and $F_t^*(c)$:

$$\dot{S}_t^*(c)=(\gamma\delta+\mu)S_t^*(c)(1-S_t(c))+\nu(1-S_t^*(c)), \tag{18}$$

$$\dot{F}_t^*(c)=\mu S_t^*(c)(1-F_t^*(c))+\nu(1-F_t^*(c)) \tag{19}$$

for $t\geqq T(c)$, with the initial condition $\dot{S}_{T(c)}^*(c)=\dot{F}_{T(c)}^*(c)=\nu$. Eq. (18) was deduced as follows. Let us ignore the forces of economic selection and technological imitation for the moment. Then, the value of $S_t(c)$ increases whenever one of the firms with unit costs higher than c succeeds in innovation. Indeed, its value increases by the magnitude equal to the innovator's capacity share. [An innovation by a firm with unit cost c or less does not affect the value of $S_t(c)$.] Since the probability of a successful innovation by one particular firm is $\nu\cdot\Delta t$ during a time interval Δt, and the capacity shares of the firms with unit costs higher than c add up to $1-S_t(c)$, the *expected* increase in the value of $S_t(c)$ can be computed as $(\nu\cdot\Delta t)\cdot(1-S_t(c))$ during the same time interval, or $\nu\cdot(1-S_t(c))$ per unit of time. If we combine this with the contributions from economic selection and technological imitation, given in eq. (12), and rewrite it in terms of the expected cumulative capacity share function, we obtain eq. (18). Eq. (19), on the other hand, can be deduced in a manner analogous to the one we employed in section 4 of the preceding paper.

Eq. (18) turns out to be a logistic differential equation of $S_t^*(c)+\nu/(\gamma\delta+\mu)$, which can be solved explicitly as

$$S_t^*(c)=\frac{1+\nu/(\gamma\delta+\mu)}{1+\dfrac{\gamma\delta+\mu}{\nu}\exp\left[-(\gamma\delta+\mu+\nu)(t-T(c))\right]}-\frac{\nu}{\gamma\delta+\mu}, \tag{20}$$

where we have employed the initial condition $\dot{S}^*_{T(c)}(c)=\nu$, which is equivalent to $S^*_{T(c)}(c)=0$. Eq. (19), on the other hand, is not an independent equation of its own. But, as is shown in appendix B, if we couple it with (20), we can still solve it to obtain

$$F_t^*(c)=1-\exp\left[-\frac{\gamma\delta}{\gamma\delta+\mu}(t-T(c))\right]$$
$$\times\left\{\frac{\gamma\delta+\mu}{\nu+\gamma\delta+\mu}+\frac{\nu}{\nu+\gamma\delta+\mu}\exp\left[(\nu+\gamma\delta+\mu)(t-T(c))\right]\right\}^{-\mu/(\gamma\delta+\nu)}, \tag{21}$$

where the initial condition $\dot{F}^*_{T(c)}(c)=\nu$ or equivalently $F^*_{T(c)}(c)=0$ was employed.

Eqs. (20) and (21) still fall short of giving us the long-run average picture of the industry's state of technology. For we still have to take into account the general tendency of unit costs to decline over time, which shifts to the left both cumulative capacity share function and cumulative frequency function. In order to neutralize this tendency, let us, as in Part I, measure the proportional gap between the unit cost of a given production method and the prevailing potential cost by a variable $z_t\equiv\ln c-\ln C(t)$ and call it the 'cost gap' of the production method in question. Then, in terms of this relative measure of efficiency, we can rewrite eqs. (20) and (21) as follows:

$$S_t^*(c)=\tilde{S}(z)\equiv\frac{1+\nu/(\gamma\delta+\mu)}{1+\dfrac{\gamma\delta+\mu}{\nu}\exp\left(-\dfrac{\gamma\delta+\mu+\nu}{\lambda}z\right)}-\frac{\nu}{\gamma\delta+\mu}, \tag{22}$$

$$F_t^*(c)=\tilde{F}(z)\equiv1-\exp\left[-\frac{\gamma\delta}{\lambda(\gamma\delta+\mu)}z\right] \tag{23}$$
$$\times\left[\frac{\gamma\delta+\mu}{\nu+\gamma\delta+\mu}+\frac{\nu}{\nu+\gamma\delta+\mu}\exp\left(\frac{\nu+\gamma\delta+\mu}{\lambda}z\right)\right]^{-\nu/(\gamma\delta+\mu)},$$

where by virtue of hypothesis (PC) we have noted the relation $t-T(c)=t+(1/\lambda)\ln c=\{-\ln[\exp(-\lambda t)]+\ln c\}/\lambda=z/\lambda$.

Eqs. (22) and (23) represent the 'long-run average cumulative capacity share function' of cost gaps and the 'long-run average cumulative frequency function' of cost gaps, respectively. They are functions only of the cost gap z and are totally independent of calendar time t! We can also calculate their density forms, respectively, as follows:

$$\tilde{s}(z) \equiv \frac{\mathrm{d}\tilde{S}(z)}{\mathrm{d}z}$$

$$= \frac{\dfrac{(\gamma\delta+\mu+\nu)^2}{\lambda(\gamma\delta+\mu)}}{\left\{\sqrt{\dfrac{\nu}{\gamma\delta+\mu}}\cdot\exp\left(\dfrac{\gamma\delta+\mu+\nu}{2\lambda}z\right)+\sqrt{\dfrac{\gamma\delta+\mu}{\nu}}\exp\left(-\dfrac{\gamma\delta+\mu+\nu}{2\lambda}z\right)\right\}^2}, \tag{24}$$

$$\tilde{f}(z) \equiv \frac{\mathrm{d}\tilde{F}(z)}{\mathrm{d}z} = \exp\left[-\frac{\nu\gamma\delta}{\lambda(\gamma\delta+\mu)}z\right]$$
$$\times\left[\frac{\gamma\delta+\mu}{\nu+\gamma\delta+\mu}+\frac{\nu}{\nu+\gamma\delta+\mu}\exp\left(\frac{\nu+\gamma\delta+\mu}{\lambda}\right)\right]^{-\mu/(\gamma\delta+\mu)}$$
$$\times\left[\frac{1+\nu/(\gamma\delta+\mu)}{\dfrac{\lambda}{\mu}+\dfrac{\lambda(\gamma\delta+\mu)}{\mu\nu}\exp\left(-\dfrac{\gamma\delta+\mu+\nu}{\lambda}z\right)}+\frac{\nu\gamma\delta}{\lambda(\gamma\delta+\mu)}\right]. \tag{25}$$

The upper part of fig. 5 illustrates a typical shape of the long-run average density of cost gaps in terms of capacity share and the lower part of fig. 5 illustrates a typical shape of the long-run average density of cost gaps in terms of firm frequency. [To save the space, we have omitted the diagrams illustrating their cumulative counterparts, $\tilde{S}(z)$ and $\tilde{F}(z)$.] The former is a long-run statistical summary of the relative distribution of the industry's total productive capacity over a (shifting) spectrum of diverse production methods, whereas the latter is a long-run statistical summary of the relative dispersion of firms over a (shifting) spectrum of diverse production methods. Together, they describe how the dynamic interactions among capacity growth, technological imitation and innovation will in the long run generate a statistical regularity out of the seemingly irregular patterns of the evolution of the state of technology. In fact, it is quite assuring that the shapes of these two 'theoretical' distributions are not unlike those of the 'empirical' distributions presented in fig. 1.

As is seen from (24) and (25), the long-run average configuration of the industry's state of technology is determined by the basic parameter values of $\gamma\delta$, μ ν and λ, each representing the force of economic selection, imitation, innovation and invention, respectively. Indeed, it is not difficult to show that,

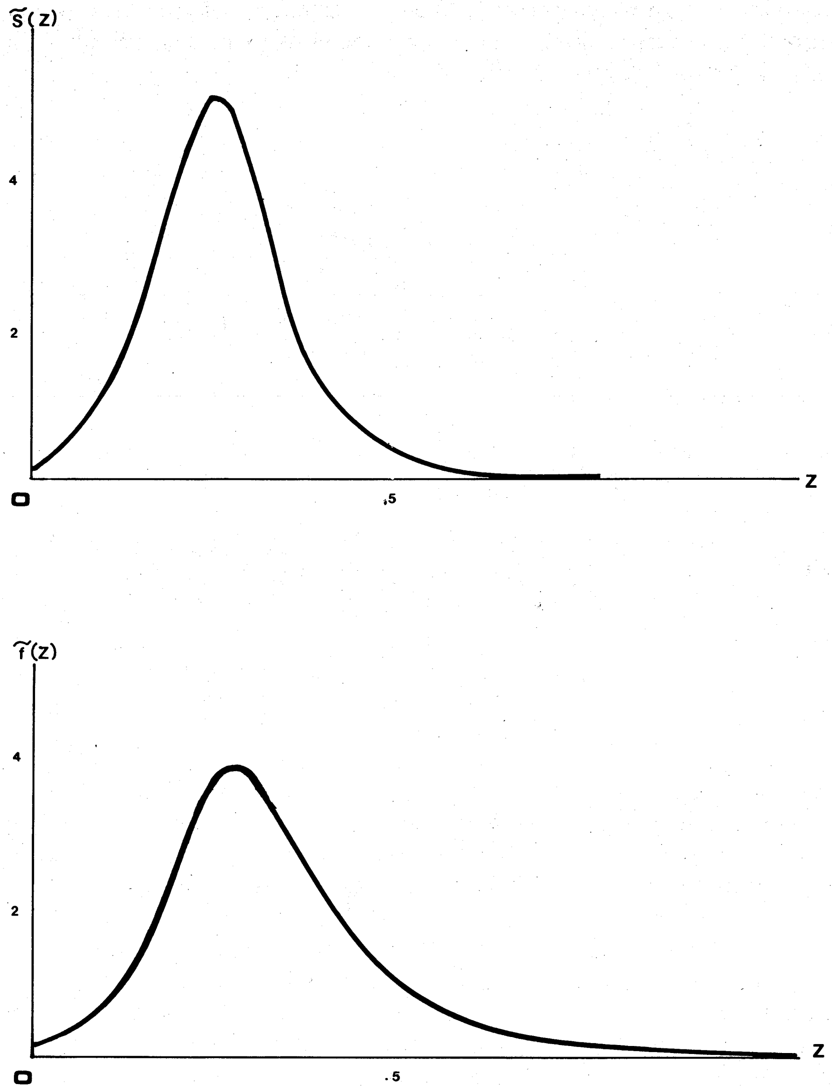

Fig. 5. The long-run average pattern of the state of technology in an industry.

other things being equal, an increase in $\gamma\delta$, μ and v tends to concentrate the dispersion of cost gaps, whereas an increase in λ tends to widen it, both in terms of the long-run average densities of capacity share and firm frequency.

What is of the primal importance is, however, not the specific results of comparative statics concerning these long-run average capacity share and frequency functions, but the general observation that a spectrum of production methods with diverse unit costs will forever coexist in the industry. Not only the fittest but also the second, third, fourth, ..., indeed, the whole range of the less fit will survive in the long run. The force of economic selection working through the differential growth rates among firms with different unit costs is constantly outwitted by the firms' imitation activities and intermittently disrupted by the firms' innovation activities. Indeed, the processes of growth, imitation and innovation will interact with each other and work only to maintain the relative configuration of the industry's state of technology in a statistically balanced form in the long run. In fact, as was already emphasized in Part I of this series of papers, it is this statistical 'equilibrium' of technological 'disequilibria' across firms, that characterizes the 'long run' of our Schumpeterian industry.

7. Empirical returns to scale

The relationship between the sizes of firms and their efficiency (or profitability) has been one of the central issues in the traditional theory of the firm and industrial organization. The question which is usually asked is: 'what general effects will the sizes of firms ... have on the efficiency attained in production and distribution?' [Bain (1968, p. 165)]. Behind this question is a static view that the size is an independent variable which functionally explains the degree of efficiency the firm attains in the form of economies or diseconomies of large-scale firms. Numerous empirical studies which try to detect the existence of positive or negative correlations between size and efficiency on the basis of individual firm data have thus been conducted in the hope that these cross-sectional correlations would reveal the underlying functional relationship between firm size and efficiency.

In the present paper, however, we have started from the premise that the unit cost of production for each firm is constant (up to the position of productivity capacity) at each point in time and hence that there exists no systematic relationship between firm size and efficiency at the level of the individual firm. The unit cost each firm has attained is the fruit of the firm's innovation and imitation activities in the past, whereas the capacity size of the firm is the cumulative result of its past growth policies whose major determinant is nothing but the profitability or the relative efficiency. Both size and efficiency have as their common cause the firm's pursuit for technological superiority in the form of innovation and/or imitation — the former as its long-run effect and the latter as its more immediate effect.

It is thus expected that this dynamic causal relation in the long run gives rise to a certain statistical relationship between capacity size and unit cost in our Schumpeterian model, even though any static relationship is by assumption precluded between them. This is indeed the case, and in fig. 6 we illustrate numerically a typical shape of the ratio between the long-run

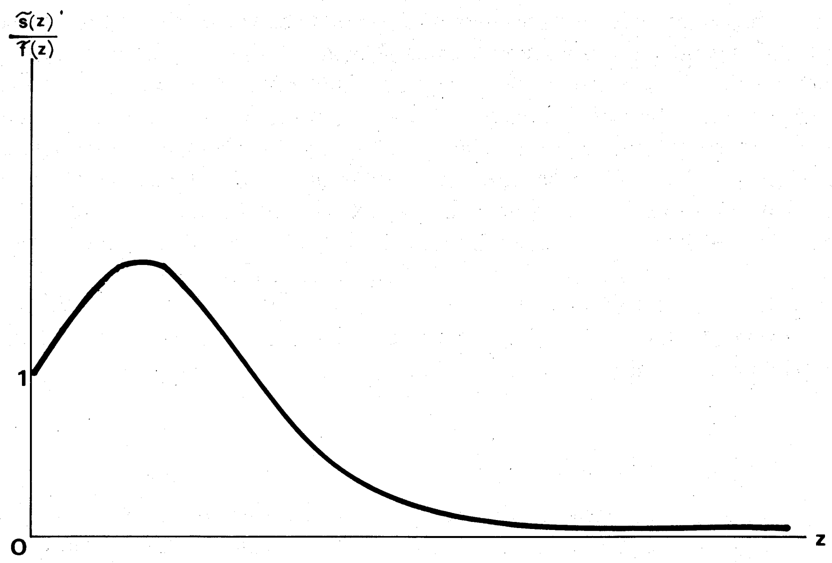

Fig. 6. The long-run average relation between efficiency and firm size.

average density of capacity share and of firm frequency, $\tilde{s}(z)/\tilde{f}(z)$. This ratio represents (approximately) the average capacity size of firms (measured in terms of the average firm size of the industry as a whole) at each value of cost gap z. It normally has a truncated bell-shape, so that there is in the long run a negative correlation between capacity size and cost gap for relatively efficient firms (i.e., for firms with relatively low values of cost gap) and a positive correlation for relatively inefficient firms. Reason for such non-monotonic correlation is not hard to come by. The firm size is nothing but the legacy of the capacity growth in the past, which was in turn governed by the relative performance of its cost conditions in the past. The most efficient firms at present are those who have recently succeeded in innovation or those which have recently succeeded in imitating the innovator. Since they are yet to exploit their good luck by rapidly expanding their capacity, the most efficient firms are unlikely to be the ones with the large capacity size. The large size firms are, on the other hand, probably those which have already passed their prime times and are currently enjoying their past success in innovation or imitation. They therefore tend to dominate in size the class of firms with modest efficiency. Finally, the firms with currently poor efficiency are likely to be small because of their relatively lower growth rates in the past.

The above explanation of the spurious relationship between size and efficiency has nothing to do with the conventional explanation based upon the static notion of economies or diseconomies of large-scale firms. If, however, empirical researchers run cross-sectional regressions of unit cost on capacity size or vice versa, they are likely to 'discover' diseconomies of scale if they restrict their data to firms which earn at least some minimum rate of profit and economies of scale if they discard high profit firms as abnormal. If they do not restrict their data set, they are then unlikely to detect any economies or diseconomies, although they are likely to 'discover' in this case that within-the-class-dispersion of unit costs increases as capacity size decreases. Needless to say, these purely theoretical predictions are very much in conformity with the results of the past empirical analyses of the relationship between size and efficiency or profitability.

Fig. 7 illustrates how this empirical relationship between efficiency and size varies as each of the basic parameters changes its value. It shows that an increase in the declining rate of potential unit cost, λ, tends to widen the range of empirical scale diseconomies and moderate the extent of both empirical diseconomies and economies of scale; that an increase in the probability of innovation, ν, tends to narrow the range of empirical scale diseconomies but at the same time slightly moderate the extent of scale economies and diseconomies; that an increase in the likelihood of imitation, μ, tends to narrow the range of empirical scale diseconomies but moderate the extent of both scale economies and diseconomies; and finally that an increase in the force of economic selection, represented by $\gamma\delta$, tends to narrow the range of empirical scale diseconomies and accentuate the extent of both scale economies and diseconomies (in fact, when $\gamma\delta = 0$, that is, when

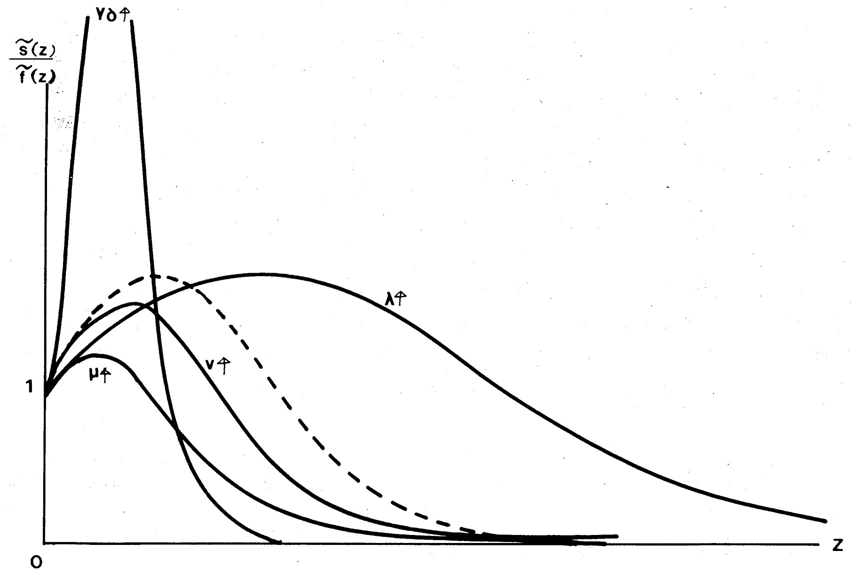

Fig. 7. Comparative statics concerning the long-run average relation between efficiency and firm size. (The base values of the parameters are $\lambda = 0.05, \nu = 0.01, \mu = 0.50, \gamma\delta = 0.50$.)

the force of economic selection is completely absent, no statistical relation should be detectable between efficiency and size across firms).

Before leaving the present section, it should be emphasized that we are not insisting on the unimportance of technological economies or diseconomies of scale. There is no doubt that they are often quite important in determining the structure of industry. All we would like to point out is the obvious possibility that, in parallel to the 'static' conditions, the industry's state of technology is also subject to the 'dynamic' forces of technological innovation, imitation and economic selection, whose effects may be strong enough to overwhelm the constraints imposed by the former. There is always a danger that mere consequences of these dynamic forces are mistaken for the existence of the static technological effects.[13]

8. Concluding remarks

The doctrine of economic selection insists, by means of the analogy to the biological theory of natural selection, that only the most efficient firms will survive the long-run competitive struggle for limited resources for capacity growth. It is this doctrine which has served the ultimate foundation of the orthodox belief in the 'rationality' of individual economic agents and the 'efficiency' of the market system as a whole, but which has seldom been formalized rigorously within the context of economic processes in which self-seeking firms, not biological species or genes, compete with each other for their survival and growth.

In the present paper, we have developed a simple dynamic model of industrial structure in which firms grow or contract (relative to others) in accordance with the success or failure of their innovative and/or imitative activities and the evolution of the state of technology of the industry as a whole is governed by the complex interplay of growth, innovation and imitation of these firms. The 'force' of the logic of economic selection has then been tested within this explicitly evolutionary model of economic process.

In the first place, we have found the paradigm of the economic selection doctrine in an artificially constructed economy in which no possibility of technological innovation or imitation is allowed to the firms. In this special environment it is not difficult to see that a firm or a group of firms which is

[13]The argument given in this section is close in spirit to that of Demsetz (1973). Mancke (1974) also developed a similar argument in the more formal manner by appealing to Gibrat-like stochastic processes. Since he did not incorporate the possibility of technological imitation nor the serial correlation of growth rates due to a successful innovation, he could only deduce a short- or medium-run positive correlation between firm profit rates and capacity sizes. His result has thus become open to criticisms by Caves et al. (1977) and Scherer (1979). Our result, on the other hand, indicates the existence of the more complex relationship between profitability and size both in the short and long run.

lucky enough to start with the most efficient production method will outgrow all the other firms and eventually dominate the whole productive capacity of the industry. Only the fittest will survive the competition and the industry will in the long run find itself in a static-equilibrium of perfect technological knowledge. Once, however, the possibility of technological imitation is brought into our model, the force of the logic of economic selection loses much of its forcefulness. It is true that even in this case only the most efficient firms will survive in the long run, and the industry will eventually settle down to a static equilibrium with perfect technological knowledge. But such long-run state is brought about, not by the success of the most efficient firms in their striving for the higher growth rate, but by the success of the less efficient firms in their efforts to imitate the most efficient ones. The blind force of economic selection is thus outwitted by the human force of imitation process. Finally, when firms are allowed to innovate in their production methods, the selective force of market competition is no longer capable of weeding out the less fit even in the long run. Not only the most efficient but also the whole spectrum of firms with diverse efficiencies will survive forever. Indeed, it has been shown that the dynamic interplay of the processes of growth, imitation and innovation will keep the industry's state of technology from settling down to the static equilibrium and reproduce in the long run a relative dispersion of efficiencies across firms in a statistically balanced form. The doctrine of economic selection itself has thus failed the 'test of survival'.

Appendix A

The purpose of this appendix is to suggest that the formal structure of our Schumpeterian dynamics developed in the main text is capable of dealing with a wide variety of industry structures. In particular, we consider here the case of a monopolistically competitive industry which consists of many firms competing with each other by producing differentiated products. Unlike the case of homogeneous product industry studied in the main text, the two causal mechanisms from successful innovation or imitation to firm growth, explained at the outset of section 2, are now both at work. Let us examine them separately.

In order to formalize the first mechanism, suppose that each firm adopts a mark-up pricing rule and sets the price of its own product p as a constant mark-up on the unit cost c; that is

$$p=(1+m)\cdot c, \tag{A.1}$$

where $m>0$ is a constant mark-up rate, which is assumed to be uniform across firms. Here, we do not analyze how this constant mark-up rate, which is sometimes called (rather tautologically) the degree of monopoly, is determined by the structure of industry. Let $\bar{c}(t)$ be the industry-wide average unit cost and $\bar{p}(t)$ the industry-wide average price, at time t. Then, in view of the mark-up relation (A.1), we have the relation between them,

$$\bar{p}(t)=(1+m)\bar{c}(t). \tag{A.2}$$

Our main hypothesis in this case is that the firm's accumulated stock of 'good-will' of customers and hence its sales volume grow at a rate which is correlated with how low its own price p deviates from the industry-wide average $\bar{p}(t)$. [See, for example, Phelps and Winter (1970) for a model of the dynamics of good-will.] Since the firm expands (or contracts) its productive capacity in accordance with the expansion of its sales volume, we can in fact suppose, as a crude first-order approximation, that the capacity growth policy of the firm with unit cost c is given by the following formula:

$$\gamma'_0-\gamma'\cdot(\ln p-\ln \bar{p}(t))=\gamma'_0-\gamma'\cdot(\ln c-\ln \bar{c}(t)), \tag{A.3}$$

by (A.1) and (A.2), where γ' is a positive constant representing the responsiveness of the growth rate of sales volume to the relative cheapness of the firm's product and γ'_0 is a constant trend growth rate of sales volume. Eq. (A.3) thus says that the growth rate of the firm is governed by the degree of its relative cost advantage, $-(\ln c-\ln \bar{c}(t))$.

The second causal chain from technological change to growth in this monopolistically competitive case is easier to formulate if the innovation is not the process innovation but the product innovation. Accordingly, let us here re-interpret the *reciprocal* of c, i.e., $1/c$, as the index of the 'quality' of the product in question. A product innovation or imitation thus amounts to an event which raises the value of this quality index $1/c$. Our hypothesis here is that the profit margin a firm can enjoy at a given point in time is determined by the relative quality of its product, which may be represented as $\ln(1/c)-\ln(1/\bar{c}(t))$. If, furthermore, we suppose that the firm's rate of capital growth is positively correlated with its current profit margin (either by its effect on the expected profitability or as the source of the internal fund for investment), we can suppose, as a very crude first-order approximation, that the capacity growth rate of the firm with the quality index $1/c$ is determined by the following formula:

$$\gamma''_0+\gamma''\cdot\{\ln(1/c)-\ln(1/\bar{c}(t))\}, \tag{A.4}$$

where $\gamma''(>0)$ and γ''_0 are given constants. This equation says that in this model of product innovation the higher the index of the product quality $1/c$ in relation to its industry-wide average $1/\bar{c}(t)$, the higher the rate of capacity growth of the firm in question.

Next, we would like to show that the two models of the firm growth given above both lead to eq. (3) for the rate of change in capacity share we deducted in section 3. This is easy to do, for a simple substitution of (A.3) into eq. (2) of the main text yields

$$\frac{\dot{s}_t(c_i)}{s_t(c_i)} = \{\gamma'_0 - \gamma' \cdot (\ln c_i - \ln \bar{c}(c))\} - \sum_{j=1}^{n} \{\gamma'_0 - \gamma' \cdot (\ln c_j - \ln \bar{c}(t))\} s_t(c_j)$$

$$= -\gamma' \cdot (\ln c_i - \ln \bar{c}(t)), \tag{A.5}$$

and of (A.4) into (2) yields

$$\frac{\dot{s}_t(c_i)}{s_t(c_i)} = \{\gamma''_0 + \gamma'' \cdot (\ln (1/c_i) - \ln (1/\bar{c}(t))\}$$

$$- \sum_{j=1}^{n} \{\gamma''_0 - \gamma'' \cdot (\ln (1/c_j) - \ln (1/\bar{c}(t))\} s_t(c_j)$$

$$= -\gamma'' \cdot (\ln c_i - \ln \bar{c}(t)). \tag{A.6}$$

Both equations are formally identical with eq. (3) of the main text. Therefore, whether the industry in question produces a homogeneous product or not, and whether, in the case of a differentiated product industry, innovation is of the process type or of the product type, the growth rate of the capacity share of a unit cost is shown to be proportional to the extent of its relative advantage over the industry average unit cost. Hence, all the results in the present paper are directly applicable to the two alternative models discussed in this appendix.

Appendix B

The purpose of this appendix is to obtain an explicit solution to the following form of differential equation:

$$\dot{x}_t = \alpha y_t(1 - x_t) + \beta(1 - x_t), \tag{B.1}$$

where

$$y_t = \frac{\gamma}{1 + \delta \exp[-\varepsilon(t - T)]} - \eta, \qquad t = T. \tag{B.2}$$

Rewrite (B.1) as

$$-\frac{\dot{x}_t}{1 - x_t} = -\frac{\alpha\gamma}{1 + \delta \exp[-\varepsilon(t - T)]} - (\beta - \alpha\eta),$$

and integrate with respect to t, we get

$$\ln(1-x_t)=\int_T^t \frac{-(\alpha\gamma)\,\mathrm{d}t}{1+\delta\exp[-\varepsilon(t-T)]}-(\beta-\alpha\eta)(t-T)+\ln(1-x_T). \quad \text{(B.3)}$$

Let $Z(t)$ represent $1+\delta\exp[-\varepsilon(t-T)]$. Then the integrand in the right-hand side of the above equation can be rewritten as

$$-\alpha\gamma\cdot\int_{1+\delta}^{Z(t)}\frac{1}{Z}\cdot\frac{1}{-\varepsilon(Z-1)}\mathrm{d}Z=-\frac{\alpha\gamma}{\varepsilon}\cdot\ln\left\{\frac{\delta}{1+\delta}+\frac{1}{1+\delta}\cdot\exp[\varepsilon(t-T)]\right\}.$$

Substituting this back into (B.3), we finally obtain

$$x_t=1-(1-x_T)\exp[-(\beta-\alpha\eta)(t-T)] \times\left\{\frac{\delta}{1+\delta}+\frac{1}{1+\delta}\exp[\varepsilon(t-T)]\right\}^{-\alpha\gamma/\varepsilon}. \quad \text{(B.4)}$$

If we identify x_t with $F_t(c)$ and set α, β, γ, δ, ε and η equal to μ, 0, 1, $1/S_T(c)-1$, $\gamma\delta+\mu$ and 0, respectively, we obtain (15) of the main text. If, on the other hand, we identify x_t with $F_t^*(c)$ and set $\alpha,\beta,\gamma,\delta,\varepsilon$ and η equal to μ, v, $1+v/(\gamma\delta+\mu)$, $(\gamma\delta+\mu)/v$, $\gamma\delta+\mu+v$ and $v/(\gamma\delta+\mu)$, respectively, we obtain (21) of the main text.

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