Fiat Money and Aggregate Demand Management in a Search Model of Decentralized Exchange: Addendum

by

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1. Introduction

In our preceding paper [1988b] we started from a detailed specification of a search-theoretic model of decentralized exchange and ended up with a textbook-like dichotomy between "Keynesian" non-neutrality and "classical" neutrality of money. When nominal prices in fiat money equilibrium are exogenously given, we showed that the economy is incapable of balancing its aggregate demand and aggregate supply and requires some kind of aggregate demand management to enhance the representative individual's expected life-time utility. When, on the other hand, nominal prices are endogenously determined (in accordance with the Nash bargaining solution) in fiat money equilibrium, we showed in two special examples that aggregate demand and aggregate supply automatically adjust themselves to each other and render the aggregate demand management superfluous. Setting aside the theoretical presumption for the built-in stickiness of nominal prices we pointed out therein, does this mean that the contribution of our search-theoretic model to macroeconomics was only to reproduce these well-known propositions at a level as elementary as that of undergraduate textbooks?

The answer to this question, however, is negative, and the purpose of this addendum is to elucidate this hastily voiced negative answer in more detail.

The present paper will continue the numbering of propositions and equations

in [1988b] and refer them without any recapitulation.

2. Fiat money, partial fiat money and autarky equilibria

Fiat money equilibrium we were exclusively concerned with in our preceding paper [1988b] is only one of a large number of possible exchange equilibria an economy may settle down. In fact, we demonstrated in an earlier paper [1988a] that the same bootstrap mechanism which generates and supports fiat money equilibrium is also capable of generating and supporting as many commodity money equilibria as the number of goods in the economy, and that when the distribution of endowments and needs is well-balanced, as is the case in the doubly symmetric example of (13a), the economy may find a comfortable niche in a barter equilibrium in which every individual seeks a direct trade with each other.[1] More important to macroeconomics is, however, a trivial form of exchange equilibrium we shall call "autarky equilibrium". We now state

<Definition>: An exchange equilibrium is said to be an "autarky equilibrium" (or "AE" for short) when no individual takes part in the economy.

We can then obtain the following proposition immediately.

<Proposition 8>: Any economy has an AE.

(Proof): Suppose all q_{ij}'s are zero. Then, all V_{ij}'s become $-\infty$, and by (3) all q_{ij}'s become in fact equal to zero, thereby justifying the initial supposition. (QED)

Before us are two extreme forms of exchange equilibrium -- fiat money equilibrium (or what we shall call "FME" for short) and AE. We know from Proposition 4 of [1988b] that a FME exists in any connected economy, and we have now learned from Proposition 8 that an AE exists in any economy. FME is a crea-

tion of the bootstrap mechanism in which every individual uses the state-issued paper as the exclusive medium of exchange simply because all the other individuals use it as the exclusive medium of exchange. In contrast, AE we have just introduced represents an absolute break-down of the decentralized coordination of exchange. No individual takes part in exchange process simply because no other individuals take part in it. It is, in other words, an outcome of the "vicious circle" or of the reversed bootstrap mechanism.

The importance of the notion of AE, however, lies not in itself but in its being the bottom end of a spectrum of equilibria, of which the FME marks the uppermost end. Indeed, our search economy generally has plenty of room for intermediate equilibria, to be introduced by

<Definition>: An exchange equilibrium is said to be a "partial fiat money equilibrium" (or "partial FME" for short) when only a fraction of individuals use the state-issued paper as money and the rest stay away from the economy.[2]

The notion of partial FME represents a partial break-down of decentralized trading coordination. A fraction of individuals are staying away from the economy either because there is no buyer of their endowment or because there is no seller of their needful good or both. As long as an economy is connected, there are both potential sellers and potential buyers, and yet their desires are not effectively communicated to each other in any decentralized manner, leaving the economy's resources partly unemployed.

Under what condition does a partial FME exist? To answer this question, consider a "subeconomy" which consists of all the individuals whose endowment and need are both included in a subset Θ of real goods. We can say that such a subeconomy is "connected" if for any i ε Θ and j ($\neq$ i) ε Θ there exists a connecting sequence of indices, k, l,.., g and h, all in Θ, such that $e_{jk} >$

0, $e_{kl} > 0$,.., $e_{gh} > 0$ and $e_{hl} > 0$. Then, as a trivial corollary of Proposition 4, any connected subeconomy must have a FME of its own, and as a trivial corollary of Proposition 8, the complement of such subeconomy must have an AE of its own. It follows that whenever an economy has a connected subeconomy it is possible to construct a partial FME with all the individuals in that subeconomy using the state-issued paper as money and all the other individuals excluded from that subeconomy staying autarky. We have thus obtained

<Proposition 9>: An economy can have at least as many partial FMEs as connected subeconomies.

How many partial FMEs does an economy have? The answer varies from zero to $\Sigma_{k=2...N-1} N!/\{(N-k)!k!\}$, depending on the economy's endowment-need structure. In the case of minimally connected economy (13b) there is no connected subeconomy. We then have only an AE and a FME and no partial FME in between. This economy is, however, an exception, and most of the economies have a large number of partial FMEs. In fact, in the case of doubly symmetric economy (13a) all the subeconomies become connected, and between AE and FME there are all together $\Sigma_{k=2...N-1} N!/\{(N-k)!k!\}$ partial FMEs!

Our search economy is thus seen to have a plethora of equilibria, which range from a full FME to an AE, often with a large number of partial FMEs located in between.

3. Pareto-rankability of partial fiat money equilibria

Consider a connected economy that has found itself in one of partial FMEs. Let us again denote by Θ a set of goods which are being exchanged through the intermediary of fiat money. We call it the "monetary domain" of a given par-

tial FME. (We can now reinterpret AE as a limiting case of partial FME with Θ being the null set and full-fledged FME as another limiting case with Θ being the whole set of real goods.) Then, by analogy to (10a) and (10b), we can calculate the steady-state frequencies of sellers and buyers in it as

$$(17a)\quad q^*_{i0} = \sum_{k \in \Theta} \frac{q^*_{k0}}{q^*_{k0}+q^*_{0i}} e_{ik} \quad \text{for } i \in \Theta \quad ; = 0 \quad \text{for } i \notin \Theta \ ;$$

$$(17b)\quad q^*_{0i} = \sum_{k \in \Theta} \frac{q^*_{0k}}{q^*_{0i}+q^*_{k0}} e_{ki} \quad \text{for } i \in \Theta \quad ; = 0 \quad \text{for } i \notin \Theta \ .$$

If we denote by M the level of per capita real balance, or more precisely, the ratio of the real balance of fiat money to the total number of individuals currently taking part in monetary exchange, we can write down the following adding-up equations for these frequencies.

$$(18a)\quad \sum_{i \in \Theta} q^*_{0i} = \theta M \ ; \quad (18b)\quad \sum_{i \in \Theta} q^*_{i0} = \theta (1-M) \ ,$$

where

$$(19)\quad \theta \equiv \sum_{i,j \in \Theta} e_{ij}$$

represents the total proportion of individuals who are currently using the state-issued paper as money. (We have $0 < \theta < 1$ in the case of partial FME, $\theta = 0$ in the case of AE and $\theta = 1$ in the case of full FME.)

Suppose then that the economy has made a transition to another partial FME with an expanded monetary domain Θ' which includes Θ as its proper subset. (The subeconomy corresponding to Θ' is connected.) Suppose also that this transition has kept the level of per capita real balance M constant by an accompanying increase in the total volume of real balance by the proportion equal to θ'/θ. A question we ask is: will this transition improve the wel-

fares of all the participants of monetary exchange? The answer is somewhat ambiguous in general but is a definite "yes" at least in the cases of doubly-symmetric economy (13a) and minimally connected economy (13b).

Indeed, the case of minimally connected economy is trivial, because it has only an AE and a full FME and the transition from the former to the latter necessarily improves every individual's welfare.

In order to examine the case of doubly symmetric economy, let Θ and Θ' be $\{1,.., n\}$ and $\{1,.., n+1\}$ respectively, without loss of generality. Note that Θ and Θ' are both connected and $\Theta \subset \Theta'$. We also have $\theta = n(n-1)/\{N(N-1)\}$ and $\theta' = (n+1)n/\{N(N-1)\}$. If we denote by q^*_{ij} and $q^{*\prime}_{ij}$ the steady-state supply-demand frequencies of partial FME with domain Θ and domain Θ' respectively, it is not hard to calculate their values explicitly as

(20a) $q^*_{i0} = \theta M/n$, $q^*_{0i} = \theta (1-M)/n$ for $1 \leqq i \leqq n$

$q^*_{i0} = 0$, $q^*_{0i} = 0$ for $n < i \leqq N$;

(20b) $q^{*\prime}_{i0} = \theta' M/(n+1)$, $q^{*\prime}_{0i} = \theta'(1-M)/(n+1)$ for $1 \leqq i \leqq n+1$

$q^{*\prime}_{i0} = 0$, $q^{*\prime}_{0i} = 0$ for $n+1 < i \leqq N$.

Because $\theta/n = (n-1)/\{N(N-1)\}$ and $\theta'/(n+1) = n/\{N(N-1)\}$, it is evident that $q^{*\prime}_{i0} > q^*_{i0}$ and $q^{*\prime}_{0i} > q^*_{0i}$ for all $1 \leqq i \leqq n+1$. Since the expected life-time utility of an i-endowed, j-consumer is given by $V_{ij} = u-2b-c/q^*_{j0}-c/q^*_{0i}$ when he is seeking a monetary exchange, we are now able to state

<Proposition 10>: In the cases of doubly symmetric economy (13a) and minimally connected economy (13b), a transition from one partial FME (or from an AE) to another (or to a full FME), which expands the monetary domain but keeps the per capita level of real balance constant, will improve the welfare of every

individual who takes part in monetary exchange.

With the multiplicity of equilibria that are often Pareto-rankable, there now emerges a "non-Keynesian" case for active government policies.

4. A "non-Keynesian" case for aggregate demand management

Monetary policy is a form of aggregate demand management which uses the stock of fiat money as its chief policy instrument. But, different from the textbook macroeconomics, we now have to distinguish two types of monetary policy in accordance with the way a change in money stock is distributed among individuals. We say that a monetary policy is "intensive" if it changes the level of per capita real balance M without affecting the monetary domain Θ and "extensive" if it does affect Θ.

When an economy is in a state of full FME, the only possible monetary policy is an "intensive" one. The state can control the level of per capita real balance M so as to keep aggregate demand and aggregate supply in equilibrium. But, as we saw in the preceding paper [1988b], this type of monetary policy becomes superfluous, at least in the two special economies (13a) and (13b), if there is a mechanism which endogenously determines all the nominal prices in accordance with the Nash bargaining solution.

If, however, a full FME has been created by self-fulfilling beliefs or what we have called bootstrap mechanism, it may also be destroyed by self-fulfilling beliefs working in the opposite direction. It may at any time collapse into one of partial FMEs or AE by an unfortunate string of pessimistic beliefs. And when the economy has actually been trapped into one of partial FMEs or AE, some form of state intervention becomes necessary to restore the full operation of the bootstrap mechanism which had supported the FME.

Indeed, the state may try to influence people's beliefs by moral persuasion, public discussions, indicative planning, and so on. More important, however, the state can also exercise an "extensive" monetary policy.

In order to see how an extensive monetary policy works in our search economy, let us again take up the case of doubly symmetric economy (13a) and assume that it has been in a state of partial FME with a monetary domain Θ = {1,.., n}. Next, suppose that the state has decided to increase the total money supply in such a way that at least one of the potential consumers of the n+1st good receives a piece of fiat money and turns himself into its effective buyer. The buyers' frequency q_{0n+1} of the n+1st good is then artificially lifted from zero and starts to activate its hitherto dormant holders. In fact, since q_{i0} has been positive for any $i \leq n$, $V_{n+1i} = u-2b-c/q_{i0}-c/q_{0n+1}$ becomes finite for any $i \leq n$ and transforms each of the inactive holders of the n+1st good into its effective seller. The sellers' frequency q_{n+10} is then raised from zero as well. This further renders $V_{in+1} = u-2b-c/q_{n+10}-c/q_{0i}$ to become finite for any $i \leq n$ and changes every potential consumer of the n+1st good into its effective buyer as soon as he has succeeded in selling his endowment. The buyers' frequency q_{0n+1} is now able to keep a positive value without any additional help from the state. An extensive increase in money supply has thus restored the decentralized coordination between buyers and sellers of the n+1st good and helped the economy move into a new partial FME (or into a full FME if n+1 = N) with an expanded monetary domain of Θ' = {1,..,n+1}. Indeed, if the state manages to keep the level of per capita real balances M stable, or if some kind of centralized price mechanism adjusts M to the Nash bargaining solution of 1/2 or its vicinity, we are able to assert by Proposition 10 that such transition will definitely improve the welfare of every participant of monetary exchange.[3]

This is what we mean by the "non-Keynesian" case for aggregate demand management, because its raison d'être has nothing to do with the "Keynesian" assumption of sticky nominal prices.[4]

Footnotes

1. See Propositions 5 and 7 in our earlier paper [1988a]. In the case of minimally connected endowment-need distribution (13b), there is no barter equilibrium.

2. There is in fact another form of partial FME which combines FME not with AE but with barter equilibrium. Other combinations may also be possible.

3. In the second special case of minimally connected economy (13b), however, the use of extensive monetary policy, which only lifts the buyers' frequencies $\{q_{0i}\}$ from zero, is not sufficient to move the economy from an AE to a full FME. It has to be accompanied by another policy which lifts the sellers' frequencies $\{q_{i0}\}$ from zero.

4. See Diamond [1984] and Cooper and John [1988] for the illuminating discussions on the relationship between multiplicity of equilibria and potential effectiveness of macroeconomic policies.

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