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**EVOLUTION OF MONEY**

by

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*<Abstract>*

   Since millennium, economists have advanced two competing theories on the evolution of money. Commodity theory asserts that money has evolved spontaneously from one of the useful commodities through a long process of barter exchanges, and cartal theory argues that money was introduced by a communal agreement or political decree or legislative action that is external to the exchange process. While economists are busy in fighting with each other, anthropologists have also developed a different story which traces the origin of money to the ancient system of gift exchange.  The purpose of this paper is to put all these stories to a critical test.

   The question of how money has evolved is intimately connected with the question of what money is. The present paper develops a simple search-theoretic model of decentralized economy which is capable of characterizing barter system, commodity money system, fiat money system, and gift system, all as different forms of its equilibrium.  It demonstrates that while barter system requires a well-balanced distribution of abilities and needs among individuals and gift system requires an infinite memory for its member, monetary system, whether it uses commodity money or fiat money, requires no such "real" condition and no such "informational" requirement to support itself as equilibrium. Money is money simply because it is used as money.  Indeed, it is this transcendence from "reality" and "information" that sets monetary system infinitely apart from both barter system and gift system, thereby throwing doubt on both economists' commodity theory explanation and anthropologists' gift exchange story.  Moreover, the pure bootstrap nature of monetary system also throws doubt on economists' cartal theory as well.  There is a fundamental limit on the power of the theory to explain the origin of money *ex post facto*.  History thus matters essentially.

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*1. Introduction: Menger, Knapp, Mauss, and the Search-Theoretic Model of Money.*

   The great thinkers of antiquity, and following them a long series of the most eminent scholars of later times up to the present day, have been more concerned than with any other problem of our science with the explanation of the strange fact that a number of goods (gold and silver in the form of coin, as civilization develops) are readily accepted by everyone in exchange for all other goods, even by persons who have no direct requirements for them or whose requirements have already been fully met. A person of the most ordinary intelligence realizes that the owner of a good will give it in exchange for one that is more useful to him.  But that every economizing individual of an entire society should be eager to exchange his commodities for small discs of metal, which ordinarily only a few men can use directly, is something that is so contradictory to the ordinary course of events that we cannot be surprised that it appears "mysterious" to even so brilliant a thinker as F. K. v. Savigny. The problem that science must solve is thus the explanation of human behavior that is *general* and whose motives do not lie clearly upon the surface.  (Menger [1871], p.315)

Since millennium economists have advanced two competing theories on the nature of money.  They

are "commodity theory of money" and "cartal theory of money."[1]  Commodity theory asserts that in

order for a certain thing to serve as a general means of exchange for commodities, it has to be a

commodity itself -- a useful thing that has an exchange value independently of its monetary function.[2]

Cartal theory, in opposition, asserts that in order for a certain thing to serve as a general means of

exchange, it needs not be a commodity itself but its use as a general means of exchange must be

approved by a communal agreement or decreed by the head of a kingdom or sanctioned by a legal

order.  These two competing theories on the nature of money have almost always been associated with

two competing views on the origin of money, though from a purely logical viewpoint they need not go

together.  Those who uphold the commodity theory tend to argue that money has evolved naturally

from one of the commodities through a long process of barter exchanges, whereas those who uphold

the cartal theory tend to argue that money was introduced into this world at a historical moment ("once

upon a time") by a communal or political or legal force that is external to the exchange process.  The

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[1] See Schumpeter [1954], especially pp. 62- 64 and 288 - 322, for the most authoritative account of this debate.  See also Monroe [1923] and Vickers [1960].   Schumpeter used the terms: metallist theory of money and cartal theory of money, or

former locates both the nature and origin of money inside of the economic sphere, and the latter its outside. Historians of monetary theory have been busy in classifying past authors on monetary matters into these two camps.[3]

Carl Menger, who laid out, along with Leon Walras and Stanley Jevons, the foundations of what is now known as neoclassical economics, was perhaps the most influential and the most sophisticated advocate of the commodity theory of money in modern times. He believed that the cartalist position was, in spite of the time-honored authority of Plato, Aristotle, and the Roman jurists who espoused it, only an easy way out of the above mentioned difficulties posed by the "mystery" of the phenomenon of money. True to his intellectual adherence to Adam Smith, what Menger tried to advance was an "Invisible Hand" account of the evolution of money. He set out to "understand the origin of money by learning to view the establishment of social procedure …as the spontaneous outcome, the unpremeditated resultant, of particular, individual efforts of the members of a society…."[4]

The key to Menger's theoretical attempt was the notion of the "salability (Absatzfähigkeit)" of goods. A good is said to have high salability if its "possession would considerably facilitate the individual search for persons who have just the goods he needs". But, not all goods are equally salable. While there is a limited demand for certain goods, that for others may be very general. And

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Metallism and Cartalism, which he borrowed from Knapp's *State Theory of Money* [1924]. Our terms -- commodity theory of money vs. cartal theory of money -- exactly corresponds to theirs.

[2] See Schumpeter [1954], p.63.

[3] According to Schumpeter [1954], the commodity theory began with Aristotle. It was advanced by Bodin, Child, Petty, Locke, Harris, Hume, Turgot, Cantillon, Scaruffi, Davanzati, Montanari, and Galiani during mercantilism, and was made an orthodoxy by classical economists, including Smith, Ricardo, and Marx. On the other hand, Schumpeter found a germ of cartal theory in Plato and saw its full development in the hands of Roman jurists. In modern times it was overwhelmed by the commodity theory and survived only in the writings of Potter, Barbon, Boisguillebert, Berkeley, James Stuart, and John Law. It was the publication of Knapp's *State Theory of Money* in 1905 that cartal theory made a strong revival. (I have, however, two objections to the classification of Schumpeter. First, his characterization of Aristotle as a commodity theorist is an downright misreading of both *Politics* and *Nicomachean Ethics*. Second, John Law cannot be classified into either camp, for he is "a class by himself" and came very close to the bootstrap theory of money we will advance in this paper. I of course need another paper to elucidate these objections.) It is also interesting to note here that as late as May of 1947, *American Economic Review* featured a debate between Benjamin Graham as a commodity theorist and Abba Lerner as a cartal theorist.

when an individual has goods with low salability, it is often difficult to obtain the goods he needs by direct barter. He may therefore find it more economical to exchange his own goods first for a more salable good even if he himself does not need it and use the latter as a temporary medium for obtaining the goods he really needs in later times. Menger then argued that "as *each* economizing individual becomes increasingly more aware of his economic interest, he is led by his *interest, without any agreement, without legislative compulsion, and even without regard to the public interest*, to give his commodities in exchange for the other, more salable, commodities, even if he does not need them for any immediate consumption purpose," and concluded that "with economic progress, we can everywhere observe the phenomena of a certain number of goods ... becoming acceptable to everyone in trade," that is, becoming money.[5] Money is thus claimed to be "a natural product of human economy," which has evolved from one of the commodities spontaneously through a long process of barter exchanges among rational individuals, without any intervention of outside authority.

In recent years, we have witnessed a growing number of theoretical works that try to solve the "mystery" of the phenomenon of money once again. They include Jones [1974], Oh [1989], Iwai [1988a, 1988b, 1996], Kiyotaki and Wright [1989, 1991, 1993], Aiyagari and Wallace [1992], Boldrin, Kiyotaki and Wright [1994], and Shi [1995]. The first objective of this paper is to present the essentials of these recent works on money and discuss their implications for the old controversy on the nature and origin of money. What distinguishes them from Carl Menger's is their extensive use of "search theory" in modeling the decentralized exchange process among rational individuals. A century has not passed in total vain. The use of search theory has enabled them to formulate the Mengerian notion of the "salability" of goods in a much more rigorous manner than Menger himself did. If, however, that were all that these recent works have accomplished, there would be no need to

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[4] Menger [1892], p. 250.

present them in this volume on "Evolution and Economics." Indeed, what we shall see in the following is an ironic fact that one of the most notable accomplishments of these works, which owe much to Menger, lies in their undermining of his very Invisible Hand account of the evolution of money.

The fundamental proposition of Menger was that when there is a commodity with high salability, every rational individual finds it more economical to use it as a medium of exchange than to seek a direct barter. But this causal order can also be reversed. We shall indeed show that once a certain good has come to be used as a general medium of exchange, the very use of it as a general medium of exchange raises its salability to the maximum at the expense of all the other goods in the economy, thereby creating the very condition for its own use as a general medium of exchange. Because of the inherent increasing-returns-to-scale nature of the decentralized matching process among searching individuals, what Menger called the salability of a good turns out to be not a given characteristic of the good itself but an endogenous variable whose magnitude is determined by the very exchange structure of the entire economy. Money is money simply because it is used as money. It is nothing but the product of a pure "bootstrap" mechanism.

What are the implications of this bootstrap nature of money for the controversy on the origin of money? In the first place, it calls in question its commodity theory explanation. It is because the high salability of a monetized good, which the commodity theorists have hypothesized as the "cause" of its use as money, may rather be an "effect" of its having already been adopted as money. Money may not have a commodity origin. Does this demise of the commodity theory imply the triumph of its rival cartal theory, which traced the monetary use of a disk of metal or a piece of paper to an "original" act of some authority outside of the sphere of exchanges? Should we accept the 1905 declaration of

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[5] Menger [1981].

Knapp that "money is a creature of law"?[6]  The answer is, however, "no."  The fact that money is able to support itself by its own bootstrap also means that money does not need any outside enforcement for its use as a general means of exchange.  Money may not have a cartal origin, either.   Knapp must share the same fate with Menger.

   It should be hastily added here that I am neither refuting the possibility that money actually evolved from a highly salable commodity nor denying the possibility that money was actually introduced by a communal agreement or an imperial decree or a legislative action.  All I am disputing is the belief, the belief that has been espoused by both commodity theory and cartal theory, that the origin of money can be determined by a theoretical consideration alone.  Commodity theory and cartal theory may each contain a truth, but it is only a half truth, not the whole truth.  No wonder that they have coexisted side by side and competed with each other from time immemorial.  Indeed, once a certain thing has started circulating as money, its bootstrap nature erases any trace of its past from itself and imposes a fundamental limit on the power of the theory to explain its origin *ex post facto*.  "History" thus matters; it matters in an essential manner in our understanding of the "mystery" of the phenomenon of money.

   In what follows I will present a simple search-theoretic model of decentralized exchange economy, which is able to formalize some of these ideas and claims much more rigorously.  The most notable feature of this model is the embarrassment of richness.  Indeed, it has a wide variety of decentralized exchange systems as its possible equilibria, such as *barter system*, *commodity money system*, and *fiat money system*.  In sections 2 - 4 I will introduce the basic ingredients of the search model, and in sections 5 - 8 I will examine the conditions for the existence of each of these different exchange systems as an equilibrium of the economy as well as the possibilities of their natural evolution.  I will

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[6] Knapp [1924]. P. 1.   The German original appeared in 1905.

demonstrate that, while a barter system requires a well-balanced distribution of abilities and needs among individuals (such as the one satisfying the so-called double coincidence of wants) to support itself as an equilibrium, a monetary system, whether it uses a useful commodity as commodity money or a useless token as fiat money, requires no "real" conditions (except what we will call the "connectedness" of the economy) to exist in this world. I will also show that there is a fundamental difficulty in the natural evolution of a commodity money system from a given "real" condition of the economy, in spite of its potential ubiquity. (A fiat money equilibrium is by definition impossible to evolve naturally.)

It turns out that our search model is able to support as its equilibrium another very important form of decentralized exchange system, thus further enhancing our embarrassment of richness. It is a *gift system*, and this will connect us to a scientific discipline wholly distinct from economics.

While economists have been fighting with each other on whether money has the commodity origin or the cartal origin, anthropologist have been elaborating an entirely different story of the origin of money. In *The Gift*, a truly classic monograph in social and economic anthropology, Marcel Mauss argued that there has never existed, either in prehistoric times or in "primitive" societies, anything that might resemble what is called a "natural" economy where people normally live in self-sufficiency and occasionally barter a deer and two beavers.[7] We human-beings are exchanging animals. We have always been exchanging things regularly with each other from time immemorial. What distinguishes "them" from "us" is only the "form" of exchanges. In pre-historic times or in "primitive" societies it is the system of gift-giving and gift-receiving that regulates the whole exchanges of goods and services among people. The system is indeed based on three simple obligations: the obligation to give, the

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[7] Mauss [1990].

obligation to receive, and the obligation to reciprocate.[8]  And it is these three obligations which set up

a perpetual cycle of exchanges among individuals, among tribes, and among generations.  The most

well-known example of such system is *kula* trade of the Trobriand Islands, though we have no space to

describe this grandiose circle of inter- and intra-tribal exchanges so beautifully recorded by

Malinowski in his *Argonauts of the Western Pacific.*[9]  What is important for our study is the claim of

Mauss and his fellow anthropologists that the system of buying and selling arose not from autarky or

barter but from the gift system.[10]  In fact, Mauss suggested that barter should be regarded as a

degenerated form of gift-giving and gift-receiving when the time span of these two acts shrinks to

zero.[11]

   Mauss' description of gift exchange as a system based on the obligations to give, receive and

reciprocate must have struck the ears of those who have ever read Axelrod's *the Evolution of*

*Cooperation* as something familiar.[12]  Indeed, the second objective of this paper is to reexamine the

evolutionary story of anthropologists – that the system of monetary exchange emerged out of that of

ancient gift exchange – from the perspective of game theory, although I am fully aware that it certainly

runs counter to the main emphasis of the anthropologists that the gift system is a total social system

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[8] See Mauss [1990], p. 39.
[9] Malinowski [1922].
[10] Note that in many of the gift exchange societies, especially among "advanced" ones, anthropologists have found
seashells, stones, metals, bones, teeth, mats, clothes, leather, cattle, tea, and many other precious objects circulating among
people and tribes.  In *kula* trade, for instance, there are a counterclockwise circulation of seashell bracelets and a clockwise
circulation of necklaces made of mother-of-pearl among the chiefs of participating of tribes.  Many anthropologists,
including Mauss, call these circulating objects "primitive money."  (Malinowski, however, dissented.)  They claim these
objects can be called "money" because they often mediate exchanges of goods and services among people and groups.
They at the same time concede that these objects are different from the modern form of money, because they are not a mere
sign or symbol of value but a sign of honor and wealth, the magical and religious symbol of rank and plenty, and because
they are not mere things, but spiritual entities still attached to the individuality of their original owners, to the history of its
past circulation, and to the collective memories of the entire tribes.  It may therefore be more accurate to characterize the
evolutionary story of anthropologists as a three stage development – a system of gift exchange without primitive money →
a system of gift exchange with primitive money → a system of selling and buying with modern money.  See, for instance,
Quiggin [1949], Dalton [1965], Einzig [1966], Codere [1968], and Grierson [1978], for detailed discussions and analyses
on the phenomena of "primitive money."
[11] Mauss [1990], p. 36.
[12] Axelrod [1984].

which encompasses not only economic but also matrimonial, religious, political, sociological, and all the other aspects of human society. Following the lead of Kocherlakota [1996] and Ishihara [1997], two important recent works on the gift system, I will study the possibilities of supporting the gift system as an equilibrium of the economy.[13] There are both a good news and a bad news to anthropologists. A good news is that, in contrast to the barter system, the gift system requires no "real" conditions to support itself as an equilibrium of the economy, thereby corroborating their claim of its prevalence in "primitive" societies. A bad news is that the gift system can sustain itself as an equilibrium only if every member of the economy can recall every other member's past actions, the past actions of their previous partners, the past actions of those previous partners' previous partners, and so on *ad infinitum*. If he or she has merely a finite-length memory, however long it is, the gift system breaks down as an equilibrium. This is in striking contrast to the system of monetary exchanges which will be shown to require no information about the members' past actions to support itself as an equilibrium. Money transcends itself not only from the "real" structure of the economy but also from its "informational" structure. There thus exists an infinite divide between the system of gift-giving and gift-receiving in "primitive" societies and the system of buying and selling in our modern monetary economy. The anthropologists' story of the evolution of money turns out to be as inconclusive as the economists' two competing stories of the evolution of money.

Yet, we human-beings are living in a full-fledged monetary economy. No matter how "miraculous" it might be from a purely theoretical standpoint, money did actually emerge on this globe in the distant past, and has since propagated itself over the entire globe. It is difficult now to find a society which organizes its economy solely by barter exchanges or by gift exchanges. The last section of this paper is a *postscript* in the double sense of the word. It was added after the conference on "Evolution and

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[13] Their models have followd an important line of research in monetary theory which emphasizes the record-keeping role

Economics," and it is concerned with the fate of the monetary system after its appearance on this globe. I will present a very brief account of the propagation process of monetary system. Its main purpose is to emphasize the difference between the logic which governs the emergence of money and the logic which governs the propagation of money – the former is about the internal development of a single economy and the latter is about the processes of selection, imitation and migration across multiple economies.

*2. The basic model of a decentralized exchange economy.*

Let us consider an economy which consists of a large number of individuals with heterogeneous combinations of abilities and needs. There are $n$ consumption goods, indexed by $i = 1, 2, …, n$, which are indivisible and come in units of size one. Each individual is endowed with a fixed ability to produce one of $n$ goods and a fixed need to consume another one of $n$ goods. Hence, a pair of the indices of goods, $(i, j)$, can be used to identify every member of this economy. (Since no individual corresponds to a twin pair *(i ,i),* we use the convention that whenever a pair of indices *(i, j)* is referred in this paper, it automatically excludes the twin pairs *(i, i)*.)

Let us denote the relative frequency of $i$-producing $j$-consumers by $e_{ij}$, and call it *ability-need frequency*. (Note that $\sum_i \sum_j e_{ij} = 1$ by constructions. By convention, the symbol $e_{ij}$ automatically excludes $e_{ii}$.) Then, the "real" structure of this economy can be summarized by *{$e_{ij}$}*, a set of ability-need frequencies, which represents how "abilities and needs," or more generally, how "technology and preferences" are distributed among its members.

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of money, such as Ostroy [1973], Ostroy and Starr [1990] and Townsend [1987, 1989, 1990].

Fully Connected Economy   Less-than-Fully-but-More-than-Minimally-Connected Economy   Minimally Connected Economy   Disconnected Economy
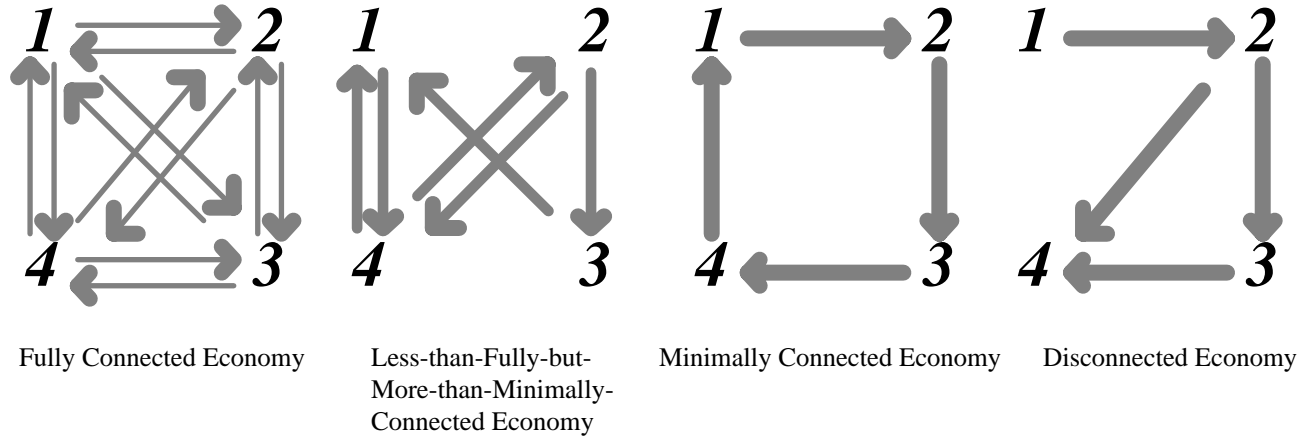
Fig. 1: Four Examples of the "Real" Structure of the Economy

There is an intuitive way to visualize this structure, and Fig. 1 shows its four examples in the case of four good economy ($n = 4$). In Fig. 1 numerals *1, 2, 3* and *4* represent the indices of goods, and *not* the indices of individuals. If there is at least one individual who produces good *2* and consumes good *3*, we draw a gray arrow from *2* to *3*, as is the case in every example. The width represents its frequency. If no producer of good *1* consumes good *3*, or it is the same thing, if no consumer of good *3* produces good *1*, no gray arrow is drawn from *1* to *3*, as is the case in the last three examples.

We now introduce the most crucial characterization of the real structure of the economy. We say that an economy is *connected* if from any index of good *i* we can visit any other index *j* by continuously following an one-way chain of gray arrows. Or, more generally, we say that an economy is *connected*, if for any *i* and *j* we have a connected sequence of strictly positive ability-need frequencies such that $e_{ih} > 0$, $e_{hg} > 0,..., e_{lk} > 0,$ and $e_{kj} > 0$.[14] Clearly, the first three examples in Fig. 1 represent three different forms of connected economy – the first one fully connected, the third one minimally connected, and the second one in between. The fourth example represents a disconnected economy.

The importance of the notion of connectedness lies in the following fact. If an economy is connected, an almighty authority could satisfy the real need of any of its members by ordering him or her to give the good he or she can produce to a second individual who needs it, who is in turn ordered to give the good he or she can produce to a third individual who needs it, and so on, until the sequence of the orders reaches an individual who is capable of producing the very good the first individual is in need of. (This divine act has just traced the above sequence, $e_{ih} > 0$, $e_{hg} > 0$,..., $e_{lk} > 0$, and $e_{kj} > 0$, in reverse order.) As long as the economy is connected, all of its members could in principle be lifted from the misery of autarky by a centralized exchange coordination. In contrast, if the economy is disconnected, no such coordination is possible even by the almighty authority. The "connectedness" is thus the minimum requirement for an association of individuals to be properly called *an* economy.

The questions we have to pose are: first, whether an autarky-breaking exchange coordination is also possible in a decentralized manner, i.e., without the support of any centralized authority, and second, if that is possible, whether such a decentralized exchange coordination is capable of evolving spontaneously from a historically given "real" structure of the economy, i.e., without the intervention of any outside authority. The first question is concerned with the existence and the second with the evolution of a decentralized exchange system. It is of utmost importance to separate these two questions, at least conceptually.

In order to give some answers to each of these questions, we now have to develop a model of the decentralized exchange economy. In recent search-theoretic works on money, there are at least two different versions of the so-called meeting technology, as is shown in Fig. 2. Oh [1989], Kiyotaki and Wright [1989, 1991, 1993] and others have supposed that the economy consists of one large trading zone, whereas Jones [1976] and Iwai [1988a, 1988b, 1996] has divided the economy into a number of

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[14] The notion of connectedness given above is closely related to that of "irreducibility" in the theory of Markov chains (see

separate trading zones, each of which is specialized to exchanges between a given pair of goods, and have let people choose trading zones to visit.[15] Both versions, however, assumed that in a trading zone people meet each other just randomly.[16] Since the basic messages of the search theory of money appear mostly independent of the precise specification of the meeting technology, I shall present here the version which is mathematically tractable and yet able to generate the most clear-cut results. It is, not unexpectedly, the one used by myself.
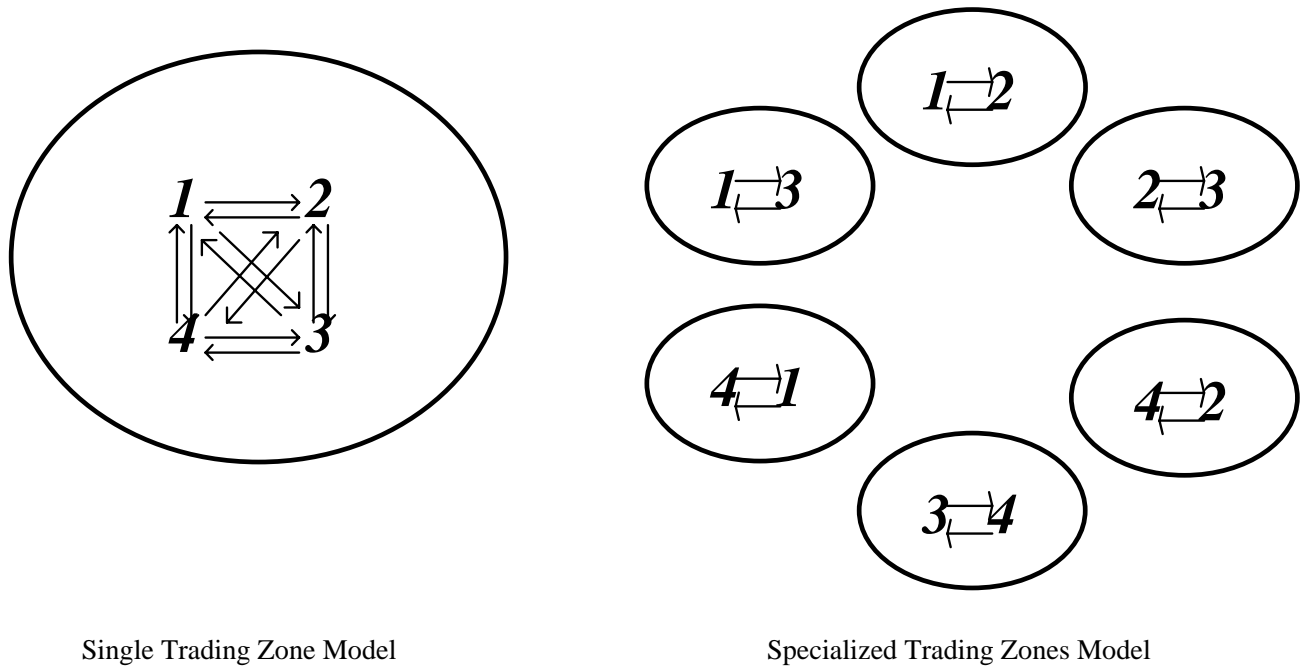


Single Trading Zone Model                    Specialized Trading Zones Model

Fig. 2 :  Two Versions of Meeting Technology

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Feller [1968]) and to that of "resource relatedness" in Arrow and Hahn [1972].

[15] To be precise, Jones himself has assumed that each trader commits him- or herself to a simple (and generally sub-optimal) search strategy and meets randomly in a single trading zone. His solution, however, turns out to be formally equivalent to Iwai's specialized trading zones model.

[16] However, there is also a variation in the assumption about the meeting probability in a trading zone. While Jones, Oh, Iwai, and the old model of Kiyotaki and Wright have basically assumed that the probability each individual meets another individual increases in proportion to the number of searching individuals, the later models of Kiyotaki and Wright have assumed away such trade externality except at the point of no searching individual. The former implies an increasing returns to scale of the quadratic order in the "aggregate" meeting probabilities, and the latter a constant returns to scale except at the zero point. Note, however, that even the latter model has an increasing returns property at least at the origin.

In this economy there are $n(n-1)/2$ trading zones, each of which is specialized to exchanges between a given pair of consumption goods $(i, j)$. (I will later introduce $n$ additional zones, to be used for exchanging real goods for fiat money in section 8 or for giving away goods for nothing in section 9.) Note that $(i, j)$ zone and $(j, i)$ zone are by assumption the same thing and that only those individuals willing to supply good $i$ in exchange for good $j$ and those individuals willing to supply good $j$ in exchange for good $i$ ever visit $(i, j)$ zone. We denote the relative frequency of $i$-supplying $j$-demanders by $q_{ij}$ and that of $j$-supplying $i$-demanders by $q_{ji}$, and call them *supply-demand frequencies.* ($\Sigma_i \Sigma_j q_{ij} \leq 1$ by construction. In what follows we again use the convention that the symbol $q_{ij}$ automatically excludes $q_{ii}$.) As we shall see later, the most crucial fact about our model of decentralized exchange process is that the set of supply-demand frequencies $\{q_{ij}\}$, which are the main determinants of the searching behaviors of individual members of the economy, in general deviate from the set of ability-need frequencies $\{e_{ij}\}$, which represent the "fundamentals" of the economy.

Now, if each trading zone is large and sparsely populated, the way people meet in it can be reasonably approximated by the so-called Poisson law. We thus assume that the probability that one of $i$-supplying $j$-demanders encounters one of $j$-supplying $i$-demanders in $(i, j)$ zone during a very short time interval is proportional to $q_{ji}$, the frequency of the latter.[17] Since we have a degree of freedom in choosing a time unit, we can set this probability equal to $q_{ji}$ per unit of time.

The supply-demand frequency $q_{ij}$ can be regarded as a quantification of what Carl Menger called the "salability" of goods. It is because the possession of a certain good, say $m$, with high $q_{im}$ would

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[17] As was suggested in the preceding footnote, this assumption corresponds to what Diamond and Maskin [1979] called the "quadratic meeting technology" in their search model. Diamond later applied this quadratic meeting technology model to the analysis of barter and monetary systems in [1982] , [1984a] and [1984b]. The prototype exchange model of Diamond, however, is a single-good barter exchange, and his "monetary economy" model presupposes a monetary transactions

facilitate the individual search for another individual who is willing to supply good $i$, thereby reducing the search cost. Likewise, a good $m$ with high $q_{mi}$ can be said to have high "purchasability," though Menger himself did not use such notion. It is because its possession would facilitate the individual's search for another individual who is willing to accept good $i$, thereby reducing the search cost.

*3.   Individual exchange strategies.*

   Now that there is no central authority to coordinate exchanges of consumption goods, every member of the economy must search for exchange partners all by oneself in order to obtain the good in need.

   Let us consider an *i*-producing *j*-consumer who has a unit of good $i$ in hand. (For the brevity of exposition, we will designate this person as "he.")   There are several alternative strategies open to him. (We will consider only pure strategies.) He can stay home and consume his own product. This is an autarky strategy. He can make a direct trip to $(i, j)$ zone and search for a mirror-symmetric individual who is willing to exchange good $j$ for good $i$. (We assume that exchange ratios are always one to one. In this model of pre-market economy, we believe we can safely leave out the issue of price-formation.) This is a barter strategy. Instead, the same individual may first make a trip to *(i, k)* zone *(k ≠ i)* and search for another individual who is willing to exchange good $k$ for good $i$. Having found such a partner and successfully exchanged good $i$ for good $k$, he then makes a second trip to *(k, j)* zone and search for another individual willing to exchange good $j$ for good $k$. This is an indirect exchange strategy which uses good $k$ as a "medium of exchange." Indirect exchange strategies using more than one media of exchange are also possible. Fig. 3 depicts some of these strategies for a *2-producing 3*-consumer.

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technology from the outset.   One of the purposes of the present paper is to deduce the very structure of the monetary transactions technology endogenously on the basis of the search-theoretic analysis of individual exchange behaviors.
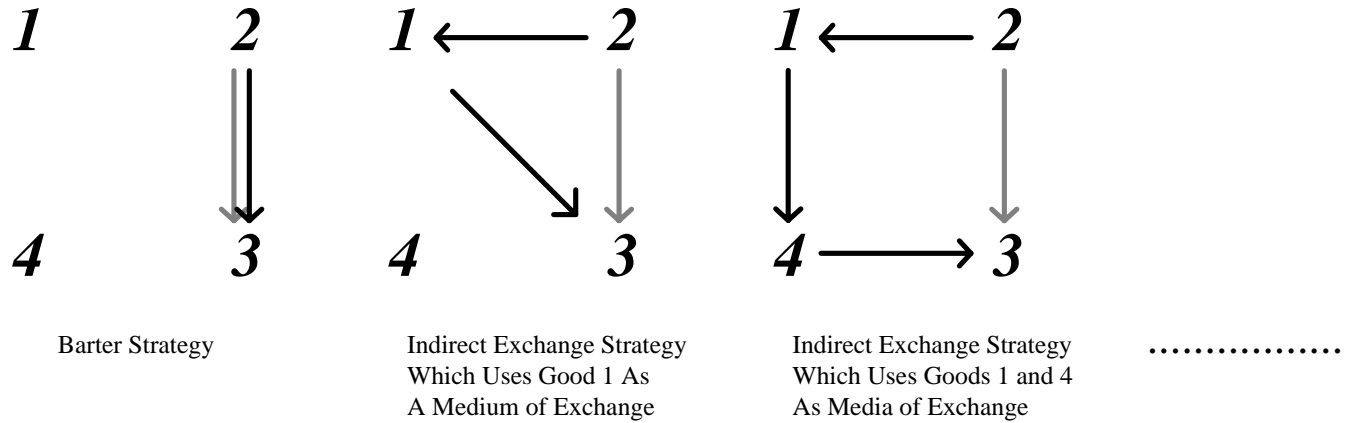
Fig. 3: Examples of Search Strategies for a 2-producing 3-consumer.

After having acquired a unit of good *j*, our *i*-producing *j*-consumer retires from the economy in order to consume it quietly at home.  For the sake of simplicity, we assume that as soon as our *i*-producing *j*-consumer has made his exit, a new *i*-producing *j*-consumer enters into the economy with a fresh unit of good *i* in hand.  Such instantaneous parent-child succession will keep the whole distribution of ability-need frequencies $\{e_{ij}\}$ constant over time.

Let us now examine the structure of the benefits and costs of each exchange strategy.  As an illustration, we first consider the case of barter strategy.  Suppose that our *i*-producing *j*-consumer has decided to barter his product *i* for his consumable *j*.  The first thing he does is to visit *(i, j)* zone and searches for a mirror-symmetric individual.  Search is of course time-consuming.  Since the probability of encountering a trading partner is $q_{ji}$ per unit of time, his expected search time can be calculated as $1/q_{ji}$.[18]  Let us assume that time is money and that its opportunity cost is *c* per unit.  Then, his expected search cost becomes equal to $c/q_{ji}$.  Having found a partner, he has to make a transaction with her.  Let us assume that the time expected to conclude a transaction is *b*.  Then, his expected transaction cost becomes equal to *cb*.  Having concluded such transaction, our consumer retires from

the economy for consumption. Let us assume that the consumption of a unit of good $j$ gives him a

utility $u$. If we subtract both the search cost and the transaction cost from this utility, we can finally

obtain our $i$-producing $j$-consumer's (undiscounted) life-time payoff of barter strategy as:

(1)  $u - c(b+1/q_{ji})$ .

Suppose next that our $i$-producing $j$-consumer has committed himself to an indirect exchange

strategy which uses good $k$ $(\neq i, \ j)$ as the sole medium of exchange. Then, he is expected to spend a

time equal to $1/q_{ki}$ in his first search, $b$ in his first transaction, $1/q_{ik}$ in his second search, and $b$ again in

his second transaction, before he can enjoy a utility $u$ of consumption. Subtracting costs from utility,

we can calculate the total life-time payoff of this indirect exchange strategy as:

(2)  $u - c(2b+1/q_{ki}+1/q_{jk})$.

In general, we can calculate our $i$-producing $j$-consumer's total life-time payoff of an indirect

exchange strategy which uses goods $k, \ l, \ ..., \ g, \ h$ as media of exchange as:

(3)  $u - c\{(\mu+1)b+1/q_{ki}+1/q_{lk}+\cdots+1/q_{hg}+1/q_{jh}\}$ ;

where $\mu$ is the number of goods used as the media of exchanges.

To make the story simple as well as complete, let us also assume that our $i$-producing $j$-consumer

suffers an infinite disutility if he fails to consume good $j$. Thus, the payoff of staying in autarky is set

to $-\infty$.[19]

The determination of the optimal exchange strategy has now become trivial. All that our consumer

has to do is to look up the above formulae and choose a strategy that generates the maximum total

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[18]  If the probability of an event is $q\Delta$ for a small time interval $\Delta$, its expected waiting time can be shown is equal to $1/q$. This is an elementary fact of Poisson process.

[19] If we set the payoff of autarky at a finite value, some of the Propositions below must be qualified and the paper's main theses will lose part of their sweepingness.

payoff (or the minimum total cost).[20]   If, however, even the maximum payoff fails to exceed -∞, he

would rather stay in autarky.  Since the number of possible strategies is finite, there is always an

optimum.   The optimal exchange strategy is determined by the way the supply-demand frequencies

$\{q_{ij}\}$ are distributed across trading zones.

   Our model of individual search behavior is this simple.  Indeed, we will see in the *Appendix* that

even if we extend the life-span of our consumers to infinity and let them solve an infinite-horizon

maximization problem (with a positive time discounting), the formal structure of the optimal exchange

strategy would retain the same mathematical simplicity.[21]


*4.  Alternative exchange systems.*

   Having thus formulated the individual exchange strategies, let us turn to the decentralized economy

as a whole and analyze its possible exchange systems.  By an *exchange system* we mean a collection of

the exchange strategies adopted by every member of the economy.  As was remarked in the

introduction, our decentralized economy allows an embarrassingly large number of such systems,

ranging from very simple to very complex.  But in the present paper we shall confine our attention

only to four relatively simple exchange systems, every one of which should occupy prominent places

in any serious study on the nature and origin of money.  They are *barter system*, *commodity money*

*system*, *fiat money system*, and *gift system*.  Barter system is perhaps the simplest exchange system, in

which everybody exchanges his or her product by direct barter; commodity money system is another

exchange system in which everybody, except its producers and consumers, uses one of the real goods

as the general medium of exchange, i.e., as a commodity money; fiat money system is an exchange

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[20] To be rigorous, we need to specify a tie-breaking rule to choose among the exchange strategies which happen to give the same payoff value.  The rule we will adopt is a lexicographic-cum-randomizing one which chooses the strategy with the shorter exchange sequence when they have different lengths and tosses a coin when they have the same length.

system in which everybody in the economy uses a state-issued fictitious good as the general medium

of exchange, i.e., as fiat money; and gift system is a *de facto* exchange system in which everybody

gives away his or her product to another as a gift and everybody receives his or her consumable from

another as a gift.

One of the fundamental problems we have set ourselves to answer in this paper is whether an

autarky-breaking exchange coordination is possible without any centralized coordinating authority.

We now restate this problem in an analytically more tractable manner: whether and under what

conditions any of the decentralized exchange systems we listed above can be supported as a Nash

equilibrium. We define a (simple) Nash equilibrium as a set of strategies each of which is optimal,

taking as given the strategies of all the other individuals in the economy. For the sake of simplicity,

we will restrict each member's strategies to pure strategies and be concerned only with steady-states of

the economy. Because of the simplicity of the information structure of the first three exchange

systems, we do not have to introduce the more sophisticated notion of sub-game perfect Nash until

section 9.

Before we proceed, we better record one (trivial) existence theorem here.

*Proposition 1*: A complete autarky is a Nash equilibrium in any economy.

*(Proof)* Suppose no one has shown up in any of the trading zones, so that every $q_{ij}$ is zero. Then,

the total search cost of any exchange strategy is infinite. Hence, everyone actually chooses to stay in

autarky, and every $q_{ij}$ becomes zero, as is supposed. (QED)

Though trivial, the above proof shows us in the most skeletal form the general structure of a Nash

equilibrium in our decentralized exchange economy. A given set of supply-demand frequencies, *{$q_{ij}$}*,

determines the optimal exchange strategies of every member of the economy, whose aggregate

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[21] The first version of this paper presented in the conference employed this infinite-horizon model.

outcomes in turn determine the actual values of supply-demand frequencies, $\{q_{ij}\}$.  Here is a circular causality, and if there is a set of supply-demand frequencies, $\{q_{ij}\}$, that is consistent with this circular causality, it constitutes an equilibrium in our model.

The autarky is the worst possible situation, from which every exchange system has to liberate itself.

*5.  On the difficulty of barter system.*

We have defined *barter system* as a decentralized exchange system in which everybody seeks to obtain their consumable by direct barter of their product.  Can such system be supported as an equilibrium?  Of course it can, and it is possible to write down a non-vacuous sufficient condition for that possibility.[22]  But a far more interesting question for the purpose of this paper is: can the barter system be *always* supported as an equilibrium?   The answer is clearly "no"!  In fact, ever since the time of Aristotle, every student of the phenomenon of money knows the following non-existence theorem:

*Proposition 2*: The barter system cannot support itself as a Nash equilibrium, unless $e_{ji} > 0$ for any $i$ and $j$ such that $e_{ij} > 0$. □

*(Proof)*:  Suppose, on the contrary, that the barter system is a Nash equilibrium even if $e_{ji} = 0$  and $e_{ij} > 0$ for some pair of $i$ and $j$.  Then, since in a barter system every active member of the economy is by definition seeking a barter exchange, we have $q_{ji} = e_{ji} > 0$ whenever $e_{ji} > 0$ and $q_{ji} = e_{ji} = 0$ whenever $e_{ji} = 0$.  This implies that the search cost of barter strategy, $c/q_{ji}$, becomes $-\infty$ for every $i$-producing $j$-consumer, so that they would stay in autarky.  Since they are active members of the economy, this is a contradiction. (QED)

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[22] The following is a sufficient condition for the barter system to be a Nash equilibrium.  Suppose that for any $i$ and $j$, $e_{ij} > 0$ and *for any* $k$ ($\neq i$, $j$) $b+1/e_{ji}) \leq 2b+1/e_{ki}+1/e_{jk}$, the n the barter system can be supported as a steady-state Nash

Of course, this is what Stanley Jevons [1875] called the "double coincidence of wants" for barter trade. It says that even if the economy is connected (and is potentially autarky-breaking for everybody), the barter system leaves some of its members forever in a state of autarky, unless the wants of its members coincide doubly, that is, unless its every member can produce the good that is wanted by another member who can produce the good he wants.



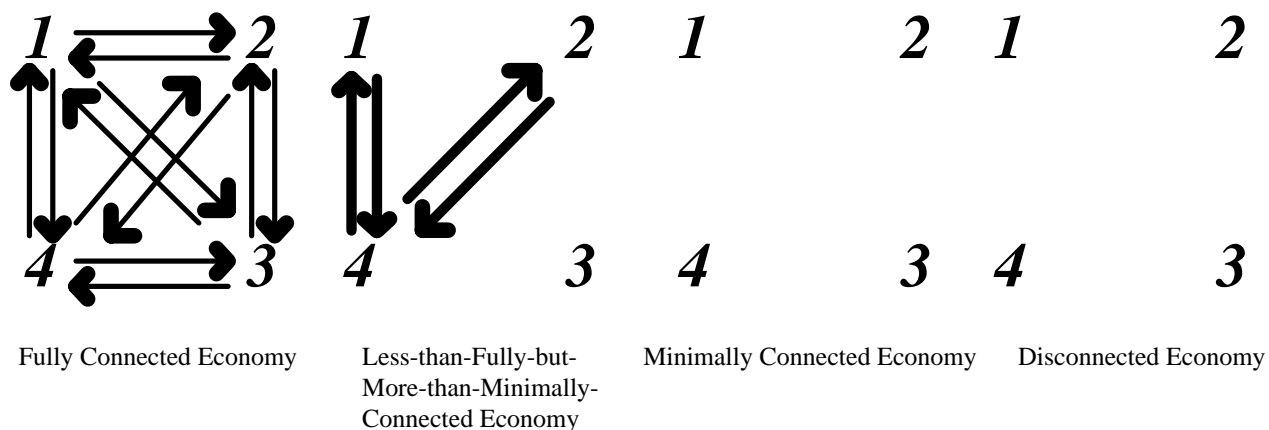| Fully Connected Economy | Less-than-Fully-but-More-than-Minimally-Connected Economy | Minimally Connected Economy | Disconnected Economy |

Fig. 4: The Exchange Structure in the Barter Exchange System

Fig. 4 represents the supply-demand structure of the barter system in each of the four examples of four-good economy in Fig. 1. A black arrow from, say, good *1* to good *2* now indicates the existence of *1*-supplying *2*-demanders in *(1, 2)* trading zone. In the case of the barter system, we can draw black arrows only between two goods which were connected both way by two gray arrows in Fig. 1. This is of course the condition of double coincidence of wants. We also have to check that no indirect exchange strategy is less costly than barter. In the first example of a fully connected economy, we can connect all four goods by two black arrows, which implies that the barter system can liberate everyone from the shackle of autarky. In contrast to this, in the third example of a minimally connected economy, nobody can satisfy the condition of double coincidence of wants, and no black arrows can

equilibrium. This is, however, an uninteresting proposition, because it is essentially the condition to prevent everybody

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be drawn in Fig. 4. The barter system fails to establish itself as a Nash equilibrium. Even though the economy is connected, thereby potentially autarky breaking, none of its members can escape from the misery of autarky. In the second example of a less-than-fully-but-more-than-minimally connected economy, the barter system saves neither *2*-producing *3*-consumers nor *3*-producing *1*-consumers and again fails to establish itself as a Nash equilibrium. Finally, in the fourth example of a disconnected economy nobody can escape from autarky, though this is the fate of its unfortunate members. Economies without barter equilibrium are not the exceptions but the rules.

The barter system is a decentralized exchange system whose supply-demand structure is completely tied down to the given "real" structure of the economy. Indeed, it is this inability to fly over the "reality" that limits the possibility of the barter system to support itself as a Nash equilibrium.

What about our second question on the evolution of the barter system? In the case of the barter system, this question turns out to be redundant. The barter system can never deviate from the "real" structure of the economy, and the condition for its existence is at the same time the condition for its natural evolution.

*6. The bootstrap nature of the commodity money system.*

Let us now turn to the analysis of the *commodity money system* – a decentralized exchange system which uses one of the real goods as the general medium of exchange, that is, as a commodity money. Let this particular good be indexed by *m*. Then, the problem we have to solve first is: under what conditions does everyone in the economy voluntarily come to use this good as the sole medium of exchange (except, of course, the one who produces it and the one who consumes it), even if it would

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from seeking indirect exchanges.

require him to spend search cost and transaction cost not once but twice?  We first state the result

which answers this question and provide its economic interpretation after.

  *Proposition 3:* A good *m* is used as the sole medium of exchange by every member of the economy

(except its producer and consumer), if the set of demand-supply frequencies $\{q_{ij}\}$ he daily observes in

trading zones satisfy the following set of inequalities for any *i* and *j* ;

(4)   $2b+1/q_{mi}+1/q_{jm} < \infty$ ;

(5)   $2b+1/q_{mi}+1/q_{jm} < b+1/q_{ji}$ ;

(6)   $2b+1/q_{mi}+1/q_{jm} < 2b+1/q_{ki}+1/q_{jk}$   for any $k \neq m$ ;

(7)   $b+1/q_{jm} < \infty$ ;

(8)   $b+1/q_{jm} \leq 2b+1/q_{km}+1/q_{jk}$   for any $k \neq m$ ;

(9)   $b+1/q_{mi} < \infty$;

(10)  $b+1/q_{mi} \leq 2b+1/q_{mk}+1/q_{ki}$   for any $k \neq m$.  □

  In spite of their formidable appearance (and their sheer number), it is trivial to give an economic

interpretation to each of the above inequalities.  The first three inequalities are concerned with the

behavior of those (unlucky) members of the economy who neither produce nor consume the monetized

good *m* at the time of their search for it.  Indeed, inequalities (4), (5) and (6) respectively say that it is

less time-consuming (hence less costly) to use good *m* as the medium of exchange than to stay in

autarky, than to barter directly, and than to use any other good as a medium.  The two inequalities (7)

and (8) in the middle are concerned with the behavior of two groups of individuals – that of the

(unlucky) members at the time of their second search for their consumable and that of the (lucky)

members who happen to produce the monetized good *m*.  They say that when an individual has good *m*

(either by a previous exchange or by a production) it is less time-consuming to exchange it directly

with his consumable than to stay in autarky and than to use some other good as a medium,

respectively. And the last two inequalities, (9) and (10), are concerned with the behavior of the other (lucky) members who can consume the monetized good $m$. They say simply that when one has a need to consume the good $m$ it is less time-consuming to seek a direct barter than to stay in autarky and than to use some other good as a medium, respectively. The formal proof of this *Proposition*, which requires some toil and labor, is given in the footnote below.[23]

   *Proposition 3* has confirmed the logic of Mengerian theory of money within our search-theoretic framework. What it says is simply that a good $m$ is used as money if its salability $q_{im}$ and purchasability $q_{mj}$ are uniformly higher than those of all the other goods in the economy, $q_{ij}$ $(i, j \neq m)$. We will, however, soon find out that this Mengerian logic can be turned upside down.

   Now, our next task is to investigate the question of whether a commodity money system can be sustained as a Nash equilibrium. Since the one half of its circular causality has already been established by *Proposition 3*, we only need to look at the other half and work out the problem of determining $\{q_{ij}\}$ as the very aggregate outcomes of the individual search activities, on the supposition that the economy has already adopted a commodity money system. We provide a heuristic (and yet rigorous) solution in what follows.

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[23] The sketch of the proof of *Proposition 3* is as follows. We first note that inequalities (4), (7), and (9) assure that no one in the economy stays in autarky. We next prove that if (8) holds for any $j$, no holders of good $m$ seek an indirect exchange. This can be done by repeatedly applying (8) to its own R-H-S and obtain a series of inequalities: $b+1/q_{jm} \leq 2b+1/q_{jk}+1/q_{km}) \leq 3b+1/q_{jk}+1/q_{kh}+1/q_{hm} \leq .....$ They of course imply that the total cost of barter, $b+1/q_{jm}$, is indeed the minimum total search cost for $m$-holding $j$-consumers. In exactly the same manner, we can also prove that if (10) holds for any $i$, no producers of good $m$ seek an indirect exchange either. Finally, we prove that if (5), (6), (8) and (10) hold for any $i$ and $j$, the total search cost of using good $m$ as the sole medium of exchange, $2b+1/q_{mi}+1/q_{jm}$, is indeed the minimum total search cost for $i$-producing $j$-consumers with $i$ and $j \neq m$. Since (5) and (6) implies that expression is smaller than the total search cost of barter as well as the total search cost of using any other good as the sole medium of exchange, what remains to be proved is only that it is also smaller than that of any longer indirect exchange. Suppose not. Then, there is a sequence of $\mu (\geq 1)$ indirect exchanges which uses goods $k,..., h$ as media, such that $2b+1/q_{jm}+1/q_{mi} > (\mu+1)b+1/q_{jk}+...+1/q_{lh}+1/q_{hi}$. Then, by applying (5) to the last term in the R-H-S, we have: $> (\mu+2)b+1/q_{jk}+...+1/q_{lh}+1/q_{hm}+1/q_{mi}$. But if we apply one of the inequalities we obtained for $m$-holding $j$-consumers at the beginning of this footnote to all the terms but the last in the R-H-S, we obtain an inequality: $2b+1/q_{jm}+1/q_{mi} > 2b+1/q_{jm}+1/q_{mi}$. This is an outright contradiction. (QED)

Anybody who ever lived in a monetary economy, and that means everybody, knows that "money buys good, goods buy money, but goods do not buy goods," even if he or she has never read a single page of economics textbook.  All we need to do here is to translate this famous dictum of Robert Clower [1965] into the determination of *{q*$_{ij}$*}* in a commodity money system.  We first illustrate this in Fig. 5 in the case of four-good economy.



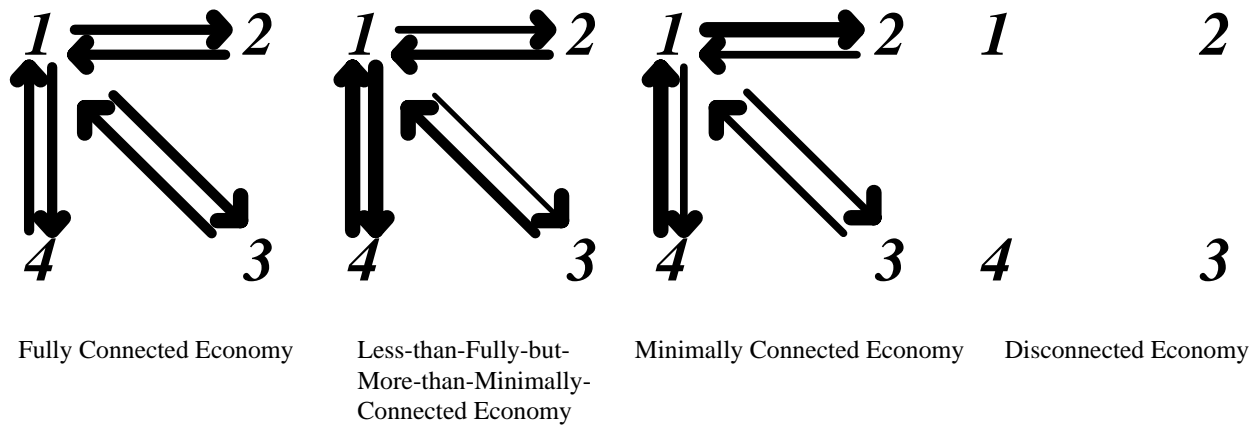| Fully Connected Economy | Less-than-Fully-but-More-than-Minimally-Connected Economy | Minimally Connected Economy | Disconnected Economy |

Fig. 5: The Exchange Structure in a Commodity Money Exchange System.
(When Good 1 is Used as a Commodity Money.)

For the purpose of the exposition let us choose good *1* as (a candidate for) a commodity money. Then, consider a *2*-producing *3*-consumer who is neither a producer nor a consumer of the monetary good *1*.  That such an individual exists is represented in Fig. 1 by a gray arrow from *2* to *3*.  Now, what the second part of Clower's dictum – "goods buy money" and the first part – "money buys goods" -- together say is that he first supplies good *2* in exchange for good *1* and then demands good *3* in exchange of good *1*.   In plain English, he first "sells" his product and then "buys" his consumable. In Fig. 5 we can represent his exchange activity by drawing a black arrow from good *2* to good *1* first and then another black arrow from good *1* to good *3*.   (If a black arrow has been already drawn there, we only add its width.)   Then, the third part of Clower's dictum – "goods do not by goods" – comes out as a mere by-product.  Even if there is a gray arrow from *2* to *3* in Fig.1, there is no black arrow

going directly from *2* to *3* in Fig. 5, implying that no *2*-producing, *3*-consumers seek to barter good *2*

for good *3*.  We repeat the same exercise for all the other existing individuals who are neither

producers nor consumers of good *1*.  As for those lucky individuals who happen to be producers or

consumers of the monetary good, they can exchange their product directly for their consumable.

Accordingly, if there is a gray arrow from or to *1* in Fig.1, we draw a black arrow directly from or to *1*

in Fig. 5.  Not every black arrow may survive, however.  If it cannot be matched by another black

arrow of the opposite direction (or if we can find a less time-consuming indirect exchange route), we

have to erase it from the graph (together with the connected black arrow), as is the case in the example

of a disconnected economy.   It turns out that if the economy is connected, as is the case of the first

three examples, we do not have to erase any of the black arrows once they were drawn in Fig. 5.   For

instance, in the case of the minimally-connected economy where we can find a continuous sequence of

gray arrows which starts from *1*, visits *2, 3*, and *4*, and then returns to *1* in Fig. 1, we can draw a

continuous sequence of black arrows from *1* to *2*, from *2* to *1* to *3*, from *3* to *1* to *4*, and finally from *4*

to *1* in Fig. 5.  Evidently, every black arrow thus drawn is matched by another black arrow with the

opposite direction.[24]

   Now, let us look at the supply-demand structure of a commodity money system as a whole the

picture of which we have just completed in Fig. 5.  In the first three examples of the connected

economy, we can find two-way black arrows between *1* and all the other indices in the economy.  This

implies that the monetary good *1* is constantly demanded and supplied in return for all the non-

monetary goods in the economy.   Even if no producers of good *2* need good *1* for consumption both in

the less-than-fully-but-more-than-minimally-connected economy and in the minimally-connected

economy, they nonetheless demand good *1* simply because it is the only good other individuals supply

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[24] A proof of *Proposition 4* to be given in footnote 26 merely formalizes this exercise.

in exchange for good *2*.   Even if no consumers of good *3* produce good *1* again both in the less-than-fully-but-more-than-minimally-connected economy and in the minimally-connected economy, they nonetheless supply good *1* they acquired in the previous exchange simply because it is the only good other individuals demand in exchange of good *3*.  Note that a black arrow pointing to *1* from, say, *2* represents the salability of good *1* against good *2* (or supply-demand frequency $q_{21}$) and a black arrow pointing from *1* to, say, *3* represents the purchasability of good *1* against good *3* (or supply-demand frequency $q_{13}$).   Note also that the absence of black arrow between the indices of two non-monetary goods, say *2* and *3,* represents the lack of their salability and purchasability against each other.  We have thus succeeded in reversing the causal order of Mengerian theory of money.  Both the high salability and the high purchasability of the monetized good against all the other goods in the economy may not be the "cause" of its use as money; they may rather be the "effects" of its use as money!

 We now have to formalize the above diagrammatic exposition in the general *n*-good economy.  First, the second and first parts of Clower's dictum – "goods buy money" and "money buys goods" -- together imply that once good *m* is used as a commodity money, any individual who is neither its producer nor its consumer first sells his product *i* ($\neq m$) in exchange for it and then buys his consumable *j* ($\neq m$) in exchange of it.  This implies that each of the *i*-producing *j*-consumers spends the first half of his life-time in the *(i, m)* trading zone as a "seller" of good *i*, and the second half in the *(m, j)* as a "buyer" of good *j*.  Since we have supposed that the probability of his encounter with one of the buyers of his product *i* in *(i, m)* zone is $q_{mi}$ per unit of time and that the probability of his encounter with one of the sellers of his consumable *j* in *(m, j)* zone is $q_{jm}$ per unit of time, his expected search time, or selling time, in the first zone is equal to $1/q_{mi}$ and his expected search time, or buying time, in the second zone is equal to $1/q_{jm}$.[25]   He is also expected to spend a time *b* in his first transaction and

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[25] Recall footnote 18.

another *b* in his second transaction.  Hence, the fraction of the life-time he is expected to spend in the

fist zone as a seller is *(1/q_{mi})/(2b+1/q_{mi}+1/q_{jm})*, and the fraction of the life-time he is expected to spend

in the second zone as a buyer is *(1/q_{jm})/(2b+1/q_{mi}+1/q_{jm})*.  Suppose that the economy has settled down

to a steady-state.  Then, Ergodic theorem of probability theory tells us that these time-series fractions

of an individual *i*-producing *j*-consumer can also be identified (with probability one) with the cross-

section fractions of all the *i*-producing *j*-consumers who are searching in *(i, m)* zone as sellers and in

*(m, j)* zone as buyers, respectively.  Hence, their steady-state frequencies in *(i, m)* zone and *(m, j)* zone

are respectively equal to *{(1/q_{mi})/(1/2b+q_{mi}+1/q_{jm})}e_{ij}* and *{(1/q_{jm})/(2b+1/q_{mi}+1/q_{jm})}e_{ij}* for any *i* and *j*

$\neq$ *m*.  If we aggregate these frequencies and do not forget to add the frequency of the consumers of

good *m* or the frequency of the producers of good *m*, we finally obtain the steady-state values of both

$q_{im}$ and $q_{mi}$ in a commodity money system.  They are

$$(11) \quad q_{im} = e_{im} + \sum_{h \neq m} \frac{1/q_{mi}}{2b+1/q_{mi}+1/q_{hm}} e_{ih} \; ;$$

$$(12) \quad q_{mi} = e_{mi} + \sum_{k \neq m} \frac{1/q_{im}}{2b+1/q_{mk}+1/q_{im}} e_{ki} \; .$$

Next, the third part of Clower's dictum -- "goods do not buy goods" – implies that in a commodity

money system no one seeks to barter a non-monetary good for another non-monetary good.  Hence, the

trading zone for any pair of non-monetary goods becomes completely empty, and we have:

(13)  $q_{ij} = 0$  for any *i* and *j* $\neq$ *m* .

Let us take a close look at equation (11) which determines the steady-state value of the frequency of

those individuals who demand good *m* in exchange of another good *i* $\neq$ *m*.  This is nothing but the

steady-state salability of the monetary good against a non-monetary good.  Its first term, $e_{im}$, is the

frequency of *i*-producing *m*-consumers, which represents the "real" demand for good *m*.  But the main

point of equation (11) is that this equation has additional terms which consist of the fractions of all the

other producers of good $i$ who are currently demanding good $m$ as the sole medium of exchange. By

the same token, equation (12) says that the steady-state frequency of those individuals who supply

good $m$ in exchange for another good $i \neq m$, or the steady-state purchasability of the monetary good

against a non-monetary good, consists not only of its "real" supply $e_{mi}$ but also of the fractions of all

the other consumers of good $i$ who are currently supplying good $m$ acquired in his first exchange as the

sole medium of exchange. These two equations say that, even if a good has originally very low

salability and very low purchasability, once a particular good is accepted as money, the very use of it

as money raises its salability and purchasability to the maximum at the expense of all the other goods

in the economy. A "bootstrap" mechanism is working here! A totally asymmetric structure of

exchange relations has thus emerged within a world of commodities. Or,

> One man is king only because other men stand in the relation of subjects to him. They, on the
> contrary, imagine that they are subjects because he is king. (Marx [1867])

Indeed, with some toil and labor relegated to the footnote below,[26] we can now prove the following

anti-Mengerian proposition in any connected economy.

*Proposition 4*: If a connected economy uses one of its real goods as the sole medium of exchange,

then at least in a steady-state all the salabilities and purchasabilites of that good become positive and

all the other salabilities and purchasabilities become zero; or if the monetized good is $m$, we have

(14)  $q_{im} > 0$ and $q_{mi} > 0$ for any $i$, and $q_{ij} = 0$ for any $i$ and $j \neq m$. □

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[26] An outline of the proof of *Proposition 4* is as follows. First of all, if the economy is connected, we can easily construct a closed loop of strictly positive ability-need frequencies, $e_{ma} > 0$, $e_{ab} > 0$, ..., $e_{yz} > 0$, and $e_{zm} > 0$, such that the set of connected indices, $m, a, b, c,..., y, z$ and $m$, contains all the $n$ indices at least once. Next, if we substitute $a$ for $i$ in (12), we have $q_{ma} \geq e_{ma}$. Since $e_{ma} > 0$, we obtain $q_{ma} > 0$. Then, if we substitute $a$ for $i$ in (11) and $b$ for $i$ in (12), we have $q_{am} \geq \{(1/q_{ma})/(1/q_{ma}+1/q_{bm})\}e_{ab}$ and $q_{mb} \geq \{(1/q_{bm})/(1/q_{ma}+1/q_{bm})\}e_{ab}$. Since $e_{ab} > 0$ and $q_{ma} > 0$, we obtain $q_{am} > 0$ and $q_{mb} > 0$ as long as $q_{bm} > 0$. If we repeat the same argument to $b$ and $c$, ...., $y$ and $z$, we obtain $q_{am} > 0$, $q_{mb}$

We then obtain one of the main propositions of this paper almost by default.

*Proposition 5*: Any connected economy can support a commodity money system as a Nash equilibrium. Indeed, it can use any of its real goods as a commodity money. □

(*Proof*). If we substitute (14) into (4) – (10) of *Proposition 3*, then all the L-H-S become finite and all the R-H-S become infinite, thereby satisfying all these inequalities. This implies that every member of the economy, except its producer and consumer, voluntarily uses good $m$ as money. It is also evident that the choice of good $m$ is arbitrary. (QED)

We have thus succeeded in establishing the "bootstrap" nature of the commodity money system. As long as the economy is connected, and this is no real restriction at all, the very process of monetary circulation creates both the general demand and the general supply (or the general salability and the general purchasability) of the monetary good at least in the long-run. Even if there is little "real" demand (or salability) for it and even if there is little "real" supply (or purchasability) of it, this "bootstrap mechanism" endows any good in the economy with all the characteristics that a money should have. A commodity money system is thus capable of sustaining itself as a Nash equilibrium without any "real" foundation to support it. If we compare Fig. 5 with Fig. 1, it is quite striking that the exchange structure of a commodity money system, represented by the configuration of black arrows in Fig. 5, is all identical in the three examples of the connected economy, in spite of the marked difference in their "real" structure, represented by the configuration of gray arrows in Fig. 1.

Money is money simply because it is used as money. Indeed, it is because of its transcendence from the "reality" that money is able to overcome the "real" constraints of the economy and make the otherwise impossible decentralized exchanges possible.

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$> 0$, $q_{bm} > 0$, $q_{mc} > 0$, ..., $q_{ym} > 0$, $q_{mz} > 0$, as long as $q_{zm} > 0$. Finally, if we substitute $z$ for $i$ in (11), we have $q_{zm} \geq$

*7. On the difficulty of the natural evolution of a commodity money system.*

A commodity money system is potentially an ubiquitous system, in the sense that it can be sustained as an equilibrium in any connected economy. But, the "potential" ubiquity should not be confused with the "actual" ubiquity. That it can exist anywhere does not necessarily mean that it does exist anywhere. On the contrary, the very "bootstrap" mechanism which empowers the commodity money system with its potential ubiquity actually works against its "natural" evolution. Its transcendence from the "reality" prevents most economies from reaching it in a spontaneous manner. This may be a self-evident implication of what we have already said in the preceding section, but we at least have to make that implication more explicit here.

Suppose we are witnessing an economy at the time of its historical "beginning." Since the only information each of its members can have is the distribution of ability-need frequencies *{eᵢⱼ}*, we suppose he bases his expectations of the supply-demand frequencies *{qᵢⱼ}* solely on this "real" structure of the economy. Can this economy evolve into a commodity money system "naturally," as Carl Menger insisted? It can -- but only if the "real" structure of the economy, represented by *{eᵢⱼ}*, has the "right" configuration.

To see how restrictive this "right" configuration is, let us look at Fig. 1 again. Of course, in the case of a disconnected economy there is no chance for the natural evolution of a commodity money, but the important point is that there is also no chance in both minimally connected and less-than-fully(-but-more-than-minimally) connected economy. For instance, even though there is at least one *2*-producing *3*-consumer in both economies, he never ventures into an indirect exchange which uses either good *1* or good *4* as a medium of exchange, because no one is willing to supply it to him in exchange for his product or demand it from him in exchange for his consumable. We already know that these two

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$e_{zm}$. Since $e_{zm} > 0$, we also obtain $q_{zm} > 0$. Hence, $q_{mi} > 0$ and $q_{im} > 0$ for all *i*. (QED)

economies fail to support the barter system as an equilibrium. But we have now seen that they also

fail to develop a commodity money system spontaneously, even though they are potentially capable of

supporting it as an equilibrium. What about the fully-connected economy? The answer depends

critically on the way ability-need frequencies are distributed among individuals. Since the example

given in Fig.1 has assumed a uniform distribution of ability-need frequencies, every individual finds it

less costly to seek a barter strategy, as we saw in section 5. It is therefore only when the distribution

of ability-need frequencies becomes much more concentrated around one particular good that there is a

chance for the natural evolution of a commodity money system.

To get a grip of this, suppose that $e_{12} = e_{21} = e_{13} = e_{31} = e_{14} = e_{41} = y > e_{23} = e_{32} = e_{34} = e_{43} = e_{24} = e_{42} = x$. (We then have an adding-up equation: $6y + 6x = 1$.) This is the most symmetrically

asymmetric "real" structure a fully-connected economy can have. It is not hard to see that if $2/y < 1/x$

$- b$, everybody in this economy comes to choose good $1$ as the sole medium of exchange, except of

course the producers and the consumers of good $1$. (Note that the above inequality corresponds to (5)

of *Proposition 3*. It is not hard to see that all the other inequalities are already satisfied.) If expected

transaction time $b$ is very small, this inequality becomes $y > 2x$. This is already a pretty stringent

condition to satisfy. And as soon as $b$ becomes non-negligible, $y$ has to be much larger than $2x$ to

satisfy this inequality. Furthermore, as the concentration of ability-need frequencies around good $1$

becomes more and more uneven, the condition for the natural evolution of a commodity money system

becomes more and more stringent. In fact, we already know that in our examples of both minimally

connected and less-than-fully-but-more-than-minimally connected economy there is absolutely no

chance for the natural evolution of a commodity money system.

It is true that the above discussions are based on a few examples and many heroic assumptions. But

their basic message is clear. Even if "*each* economizing individual becomes increasingly more aware

of his economic interest," he is *not* likely to be "led by his *interest, without any agreement, without legislative compulsion, and even without regard to the public interest,* to give his commodities in exchange for the other, more salable, commodities, even if he does not need them for any immediate consumption purpose."[27] Money is not necessarily a natural product of human economy.

*8. The "pure" bootstrap nature of the fiat money system.*

   The bootstrap nature of the commodity money system, we demonstrated in section 6, immediately suggests us the possibility of circulating as money a totally useless disk of base metal or a totally useless piece of paper or a mere acknowledgment of the ownership of a large round stone sunken deep in the sea.  This then leads us to the third form of decentralized exchange system – fiat money system. We have defined *fiat money system* as a decentralized exchange system in which everybody uses a totally useless token issued by the state as the exclusive medium of exchange.[28]
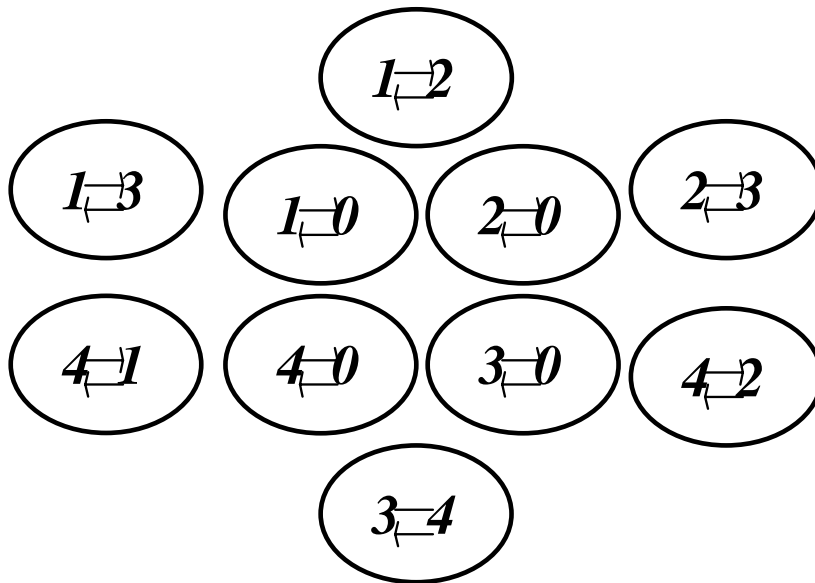


Fig. 6 :  Trading Zones When the State Issues Useless Tokens

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[27] Menger [1871].
[28] This section draws partly from Iwai [1988b].

Let us designate the state-issued token by an index *0* and expand the set of goods in the economy to *{0, 1, 2, …, n}*. As is seen in Fig. 6., we also enlarge the set of trading zones by adding *(0, 1), (0, 2), …, (0, n)* zones, each of which is specialized to exchanges between a useless token and one of the "real" goods. Since the token cannot be consumed by any private individual, we have $e_{i0} = 0$ for any $i$; since the token cannot be produced by any private individual, we have $e_{0i} = 0$ for any $i$. Let us suppose that the state sets the value of each piece of token equal to one and fixes its stock level once and for all at the beginning of the history. We denote the level of its "per capita" stock by *M*. Since the value of money is one, this represents the "real" per capita supply of useless tokens.

The question we now ask is: can the state-issued useless token circulate as "fiat money" – as the exclusive medium of exchange which is handed over from one member of the economy to another and forever staying in the sphere of exchange? In order to answer this question, we again have to go through two complementary exercises. First, we have to deduce the conditions on the supply-demand frequencies, *{q_{ij}}*, which induce every individual to use the useless token voluntarily as the sole medium of exchange. Second, we have to determine the values of these supply-demand frequencies as the very aggregate outcomes of all the individuals' search processes, on the condition that they all use the useless token as the sole medium of exchange. Fortunately, we can invoke most of the propositions about the commodity money system, if we add a few obvious modifications.

First, we have:

*Proposition 6:* Every member of the economy uses a totally useless token *0* as the sole medium of exchange, if the set of supply-demand frequencies *{q_{ij}}* satisfy the following set of inequalities for any $i$ and $j$ $(\neq 0)$;

(15)   $2b + 1/q_{0i} + 1/q_{j0} < \infty$ ;

(16)  $2b+1/q_{0i}+1/q_{j0} < b+1/q_{ji}$ ;

(17)  $2b+1/q_{0i}+1/q_{j0} < 2b+1/q_{ki}+1/q_{jk}$    for any $k \neq 0$ ;

(18)  $b+1/q_{j0} < \infty$ ;

(19)  $b+1/q_{j0} \leq 2b+1/q_{k0}+1/q_{jk}$   for any $k \neq 0$. $\square$

All the five inequalities have self-explanatory economic interpretations.  The first three are concerned with the first stage of the search activity, in which everyone is selling his product in exchange for the state-issued token, and the last two are concerned with the second stage of the search activity, in which everyone is buying his consumable in exchange of the state-issued token.  Indeed, inequalities (15), (16) and (17) respectively say that it is less time-consuming (and less costly) to use the token $0$ as the exclusive medium of exchange than to stay in autarky, than to barter directly, and than to use any "real" good as a medium.  And inequalities (18) and (19) respectively say that after an individual has acquired the token $0$ in exchange of his product, it is less time-consuming (and less costly) to exchange it directly with his consumable than to stay in autarky and than to use any "real" good as a medium.

Next, we can also determine the steady-state values of ability-need frequencies $\{q_{ij}\}$ in the fiat money system in exactly the same way as in the commodity money system.  If we keep in mind that no one consumes nor produces the state-issued token, or $e_{0i} = e_{i0} = 0$ for any $i,$ a mere substitution of the index $0$ for $m$ leads us to the following expressions in the case of fiat money system:

(20)  $q_{i0} = \sum_{h \neq 0} \dfrac{1/q_{0i}}{2b+1/q_{0i}+1/q_{h0}} e_{ih}$ ;

(21)  $q_{0i} = \sum_{k \neq 0} \dfrac{1/q_{i0}}{2b+1/q_{0k}+1/q_{i0}} e_{ki}$ ;

(22)  $q_{ij} = 0$   for any $i$ and $j \neq 0$ ;

Equation (20) says that in *(0, i)* trading zone the demand for the state-issued token consists of the fractions of all the producers of good *i* who are currently demanding the token as money; and equation (21) says that in the same *(0, i)* trading zone the supply of the state-issued token consists of the fractions of all the consumers of good *i* who have acquired the token as money in the previous exchange and are currently supplying it in exchange for good *i*. Finally, equation (22) says that all the zones exchanging a pair of "real" goods are completely vacated. We are again witnessing the work of a bootstrap mechanism! Indeed, the bootstrap mechanism in the fiat money system is much purer than that in the commodity money system. Even if there is no one "really" demanding and no one "really" supplying a state-issued token, that is, even if $e_{i0} = 0$ and $e_{0i} = 0$ for any *i,* the very decentralized exchange process which uses it as money raises its demands (salabilities) and supplies (purchasabilities) to the maximum at the expense of all the "real" goods in the economy.

In the case of fiat money system, we need one more equation to complete the model. It is the equation which balances the total demand and the total supply of the stock of money. Since each member of this economy holds the state-issue token solely for the purpose of transaction and carries only one piece of it at a time, the existing stock of money *M* determines the total frequency of individuals who hold money and are able to demand their consumables at each point in time. Hence, we have the following "effective demand equation":

(23)  $\sum_i q_{0i} = M$ .

Note that in the fiat money system the "goods do not buy goods" equation (22) reduces the adding-up equation of supply-demand frequencies: $\sum_i \sum_j q_{ij} = 1$ to a degenerate form of $\sum_i q_{0i} + \sum_j q_{j0} = 1$. Hence, we also have a complementary equation which determines, as a residual, the total frequency of individuals who can supply their products at each point in time:

(24) $\sum_j q_{j0} = 1 - M$ .

Equation (23) says that if $M > 0$, at least one member of the economy can appear in *(0, i)* zone as a supplier of money (or equivalently as a buyer of good *i*), or $q_{0i} > 0$ for some *i*. And equation (24) says that if $M < 1$, at least one member of the economy can appear in *(0, j)* zone as a demander of money (or equivalently as a supplier of good *j*), or $q_{j0} > 0$ for some *j*.



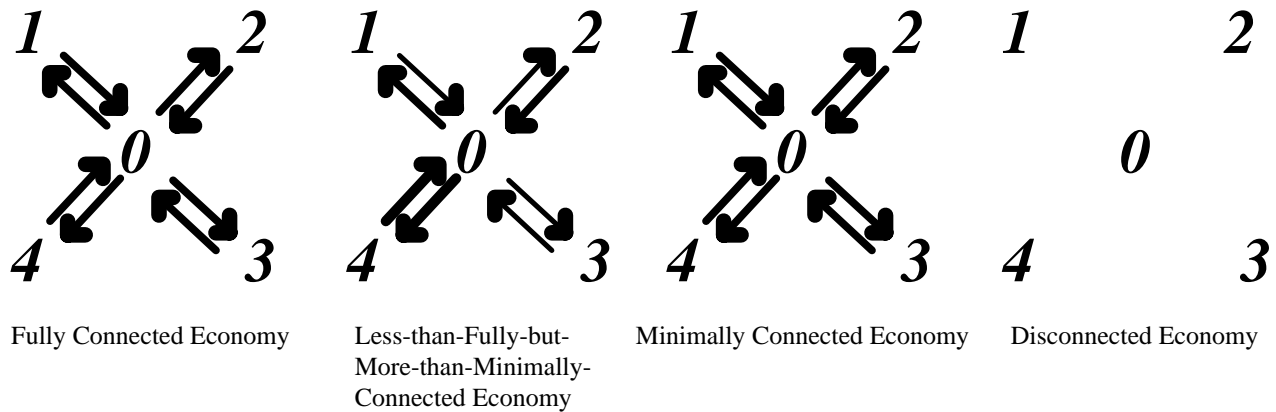| Fully Connected Economy | Less-than-Fully-but-More-than-Minimally-Connected Economy | Minimally Connected Economy | Disconnected Economy |

Fig. 7 : The Exchange Structure in the Fiat Money Exchange System

Fig, 7 then visualizes some of the above discussions. It has added the index *0* of the state-issued token in the middle of the four indices of "real" goods. We have now inscribed in it the supply-demand structure of the fiat money system. As in the case of a commodity money system, if there is a gray arrow between a pair of real (and hence non-monetary) goods, say, from *2* to *3* in Fig. 1, we draw a black arrow from *2* to *0* and another black arrow from *0* to *3* in Fig. 7. (We, however, have to erase them, if one of them is not matched by another black arrow of the opposite direction.) The first black arrow represents the "goods buy money" part, the second black arrow the "money buys goods" part, and the absence of the direct black arrow from *2* to *3*, in spite of the existence of a gray arrow in Fig. 1, represents the "goods do not buy goods" part of Clower's dictum. It should be evident just by looking at the diagrams that in the first three examples of the connected economy we can draw two-way black arrows between *0* in the middle and all the real goods in the economy. This means that if

the stock of the state-issued token is positive (but does not saturate the economy completely), the totally useless token is constantly supplied and demanded in return for all the real goods in the economy at least in a steady-state. In the case of a disconnected economy, we again fail to draw any solid arrows. This quasi-visual argument can be made completely rigorous,[29] and we have:

*Proposition 7:* Suppose that the "real" part of the economy is connected and that $0 < M < 1$. Then, at least in a steady-state every demand (or salability) for and every supply (or purchasability) of the state-issued token become strictly positive, and all the other salabilities and purchasabilities become zero, or

(25) $q_{i0} > 0$ and $q_{0i} > 0$ for any $i$, and $q_{ij} = 0$ for any $i$ and $j \neq 0$. $\square$

This *Proposition* has turned Mengerian theory of money completely upside down. Since the fiat money has by definition no "real" demand and no "real" supply, its high salability and high purchasability (relative to those of all the "real" goods in the economy) should be the pure "effect" of its use as money.

As in the commodity money system, once this pure anti-Mengerian *Proposition* is established, we can prove the "bootstrap" nature – this time, the pure bootstrap nature -- of fiat money almost by default.

*Proposition 8:* Any connected economy can support a fiat money system as a Nash equilibrium, if the per capita stock of the state-issued fiat money $M$ satisfies $0 < M < 1$. $\square$

*(Proof):* Substitute (25) into (15) - (19) of *Proposition 5*. (QED)

We can now assert, more forcefully than before, that money is money simply because it is used as money. Fiat money is the product of a pure bootstrap mechanism.

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[29] If $0 < M < 1$, we have by (23) and (24) $q_{0i} > 0$ for at least one $i$ and $q_{i0} > 0$ for at least one $i$. Then, we can apply the argument in footnote 26 to (20) and (21) and prove $q_{0i} > 0$ for any $i$ and $q_{i0} > 0$ for any $i$. (QED)

If we compare Fig. 7 with Fig. 1, it is quite striking that the exchange structure of fiat money system is, just like that of commodity money system, all identical in the three examples of the connected economy, in spite of the marked difference in their real structure. What is equally striking is, however, that, unlike that of commodity money system represented in Fig. 5, the exchange structure of fiat money system in Fig. 7 now shows a complete symmetry with respect to the real goods in the economy, in spite of the marked asymmetry in some of their underlying real structures. Indeed, there remains no trace of the underlying real structure of the economy once it has started circulating the state-issued token as fiat money.

What does this all mean to the controversy on the nature and origin of money? It of course means the complete demise of the commodity theory of money, for the circulation of fiat money severs money from any connections with useful commodities. But the important thing is that it also means the demise of the cartal theory of money as well. Because, as we have now seen, the bootstrap mechanism of money can endow a totally useless token with the power to circulate as money, even without any state legislation and even without any state guarantee of convertibility. In this sense, the term "fiat money" is misleading. It is not the fiat of the state that circulates a useless token as money. The only place the "fiat" enters into the scene is the designation of the material (as well as its quantity) that serves as money.

Before we leave this section, we better note a way to incorporate macroeconomic policies into our model of fiat money system. If the state controls the level of real monetary balance $M$, it constitutes the monetary policy of this system. If the state controls the values of $e_{0i}$ and $e_{i0}$ by entering into the economy as a buyer and a seller of the real goods (in exchange of the fiat money it issues), it constitutes the fiscal policy of this system. We can then do an exercise in elementary macroeconomics simply by working out the impacts of these policies on the well-beings of the various members of the

economy.  Needless to say, however, our search model is still too primitive to do any serious

macroeconomics beyond that exercise.


*9. The gift system and its informational requirement.*[30]

Let us now cross the traditional boundary of economics and move to the world of anthropologists –

the system of gift-giving and gift-receiving.  We have defined *gift system* as a *de facto* exchange

system in which every member of the economy gives his product as a gift to another member of the

economy who needs it and every member of the economy receives his consumable as a gift from

another member of the economy who produces it.  Although the model of the economy we have so far

employed is designed especially for the analysis of real exchange process and by no means the

simplest model of gift-giving and gift-receiving process, we keep employing it in this section for the

sake of comparability.  It needs one alteration in order to serve as a model of the gift-giving process.

Let us remove the state-issued token or fiat money from our picture of the economy.  We, however,

retain $n$ trading zones associated with it and transform them into "gift-giving" zones.  For instance, *(0,*

*i)* zone now functions as the zone in which individuals give away a unit of good $i$ to anyone who wants

it.  (Hence, *0* stands literally for "nothing.")  The supply-demand frequencies, $q_{i0}$ and $q_{0i}$, now

represent the number of the individuals who are looking for someone to give a unit of good $i$ as a gift

and the number of the individuals who are looking for someone who gives them a unit of good $i$ as a

gift, respectively.

Our interest in the gift system lies in the following observation.  Imagine an economy every member

of which feels a strong moral obligation to make a gift to its needy members.  This is of course the

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[30] This section owes to Kocherlakota [1996] and Ishihara [1997].  Their model settings are, however, different from ours.
Kocherlakota has examined a Samuelson's overlapping generation model, Townsent's turnpike model, and Kiyotaki-
Wright's search model, whereas Ishihara has used a simplified overlapping generation model.

world of anthropologists. For instance, after having produced a unit of good $i$, an $i$-producing $j$-consumer visits $(0, i)$ zone to meet one of the consumers of good $i$ in order to make her a gift of a unit of his product. After having done such an honorable act, our consumer-producer visits $(0, j)$ zone with a peace of mind and waits for an encounter with a producer of good $i$ who is willing to make him a gift of a unit of good $j$. If every member of the economy follows such gift-giving and gift-receiving practice, this virtuous economy can extricate itself from the misery of autarky. What should be noted is a fact that as far as the set of supply-demand frequencies $\{q_{ij}\}$ is concerned there is a complete correspondence between the gift system we have just described and the fiat money system we discussed in the preceding section. This leads us to:

*Proposition 9*: The gift system and the fiat money system can completely replicate each other's transfers of real goods in the economy.[31]

An economy we want to model in this paper is, unfortunately, much baser than the world of anthropologists. We do not want to rely on the sense of obligation to transfer real goods among its members. If there is an act of gift-giving, we want it to be motivated solely by the giver's self-interests. In other words, we want to work in the paradigm of economists. So, the questions we must ask in this section are the same as before. Can a gift system be sustained as a Nash equilibrium? If it can, can it be *always* sustained as an equilibrium? If not, what is the condition for its sustainability? This time, however, our analysis must be more explicitly game-theoretic than in the previous sections, and our equilibrium concept must be that of sub-game perfect Nash. It is not because we are working on the model of a base society, but because the actions taken by each individual in the gift system depend critically on the *information structure* of the economy.

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[31] This *Proposition* should be contrasted with Kocherlakota [1996] who finds that in Kiyotaki-Wright model the transfer of goods in the gift system is Pareto-superior to that in the fiat money system. This result is due to the fact that in Kiyotaki-

This paper will deal with only two structures -- the case of *perfect information* and the case of *bounded information*. It is, however, possible to incorporate more complex information structures into our model, as Ishihara [1997] has recently done in a different model-setting. In the case of perfect information, any member of the economy is assumed to have a full and true memory of all the past events, or equivalently have an access to an infinite-length record of every other member's past actions, the past actions of their previous partners, the past actions of those previous partners' previous partners, and so on *ad infinitum*. In the case of bounded information, every member is assumed to have only a finite-length record which can trace the previous action of his current partner, the previous action of that partner's previous partner, and so on, but only up to the $l$-th past partner. We assume $0 \leq l < \infty$. An important limiting case is the case of $l = 0$ – the case where no one has any information about the past acts of even his current partner. In general, a strategy of each member of the economy is a function of the available set of information. A sub-game perfect Nash equilibrium is then defined as a set of strategies such that the action prescribed by each strategy after any possible history of the economy is optimal, taking as given the actions prescribed by the strategies of all the other individuals. As before, each member's strategies are restricted to pure strategies and only the steady-states of the economy are considered.

Now, in this gift system everybody has a temptation to become a beggar. If he could receive a gift without himself giving a gift to another, he would be doubly better off. Because he could then consume his own product (though its utility is assumed to be $-\infty$ in this paper), and he could save the search cost of finding a person to make a gift. The problem is to find out a decentralized incentive system which can effectively stem off such temptation.

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Wright model an option for barter trade remains even in a fiat money equilibrium, thereby weakening the punishment of deviations.

For this purpose, let us consider the following "penal code." "Always give your product to a needy individual who has never broken the penal code in the past, but never give your product to any individual who has broken the penal code at least once in the past." Note that this penal code has a self-referential structure which "ostracizes" those who deviate from the gift-giving practice, those who sympathize with those deviants, those who sympathize with those sympathizers of the deviants, and so on. If every member of the economy follows this penal code as their strategy, we of course have a gift system. First, a good news.

*Proposition 10:* If every member of the economy has perfect information, then any connected economy can support a gift system as a sub-game perfect Nash equilibrium. □

*(Proof)*: Suppose that everybody follows the "ostracizing" penal code as his strategy and suppose that the economy has already settled down to a steady-state. Then, by *Proposition 7* the economy replicates the steady-state of the fiat money system, and we have:

(26)   $q_{io} > 0$  and $q_{0i} > 0$  for any $i$ , and $q_{ij} = 0$  for any $i$ and $j \neq 0$.

This effectively closes off any kind of exchange activities – barter, monetary, and what not – from everybody's choice set. There thus remain only three strategic options open to each member of the economy – the first one is to make a gift of his product to a needy member who has kept the penal code and then wait for a gift from another, the second to make a gift of his product to a needy member who has broken the penal code and then wait for a gift from another, and the third to consume his own product and then wait for a gift from another. But if every member of the economy except he follows the penal code, the choice of the second and the third option would invite an immediate ostracism and condemn him to the state of autarky. His optimal choice is then to stick to the penal code and enjoys the same payoff as in the fiat money system. Since this is true for any individual and at any point in

time, the collection of the penal code strategies by all members of the economy constitutes a sub-game perfect Nash equilibrium. (QED)

What *Proposition 10* says is that any connected economy can in principle break the autarky and overcome the difficulty of barter system without any general medium for exchange. What is needed is only a penal code which tells everybody to ostracize deviants and their sympathizers forever from the gift system. In this sense, *Proposition 10* corroborates the claim of anthropologists that it is not the barter system but the gift system that has universally regulated the exchanges of goods and services in the so-called "primitive" societies. After the now classical work of Axerlod [1984] on the evolution of cooperation, it is not difficult to tell a nice story about the evolution of the gift system.

Does this mean that money is superfluous as a decentralized coordination device? The answer is, however, "no," for we also have the following bad news.

*Proposition 11*: If every member of the economy has only a bounded information, then even a connected economy cannot support a gift system as a sub-game perfect Nash equilibrium. □

*(Proof)*[32]: First consider the case of $l = 0$. It is then obvious that no one in a gift-giving zone can base his decision on whether an individual he meets there has made a gift or not. Next, consider the case of $l = 1$. Then, no one in a gift-giving zone can base his decision on whether his partner's previous partner has made a gift or not. Knowing that the next partner's decision cannot depend on whether the current partner has made a gift or not, no one can base his decision on whether his current partner has made a gift or not. This then implies that no one can base his decision on whether both his current partner and that partner's previous partner have made a gift or not. We can repeat the same backward-induction type argument for any $l$ as long as it is finite, and show that no one can base his strategy on whether others have made a gift or not in the past. Hence, there is no way to design a

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[32] The basic idea of this proof owes to Ishihara [1997], part (3) of Proposition 3.

system of strategies which can punish a deviant in every possible history of the economy, and everybody succumbs to the temptation to become a receiver of a gift without having made a gift before. (This also implies the breakdown of the configuration of $\{q_{ij}\}$ given by (20) -- (22). But this only strengthens the incentive for deviation.) The gift system cannot be supported as a sub-game perfect Nash equilibrium. (QED)

The implication of *Proposition 11* is quite disappointing. Because it says that the gift system can sustain itself among self-seeking individuals only in a tribal society or in a small village or in a closed community, where everybody knows each other (and each other's every ancestor) perfectly well. This is of course the paradigm of anthropologists, although their model human-beings are not necessarily *homo economicus*. As soon as the economy becomes more open, people begin to lose track of each other's past actions. Then, the temptation for deviant behaviors increases, and the tight-network of the gift system is sure to start unraveling. And, once we are in what Friedrich Hayek called the "great society" where everybody is essentially a stranger to each other, there is no chance for the survival of the gift system. We are back to where we started.

Let us look again at the decentralized exchange systems treated in the preceding five sections from the informational perspective. It is now easy to see that none of them – barter system, commodity money system, and fiat money system -- depends on the informational structure of the economy. They are consistent even with the extreme *l = 0* case of the bounded information structure. You do not have to know who your trading partner is when you conduct an exchange by direct barter or through the intermediacy of money; all you have to know is whether your trading partner really has the good or money you want. (It is for this reason that we only needed the notion of simple Nash equilibrium to characterize these exchange systems.) For the sake of completeness, we record here:

*Proposition 12*: Neither fiat money system, nor any of commodity money systems, nor barter system requires any member of the economy to know each other's past actions in order to support itself as a sub-game perfect Nash equilibrium. □

Even if the gift system and the fiat money system are alike in their independence from the "real" structure of the economy as well as in their effects on the transfers of real goods, they are totally unlike in their requirement of the informational structure of the economy.  While the former needs the help of infinite memory of every member of the economy to support itself, the latter needs none.  Indeed, what circulates money as money is not the shared communal memory but the shared expectations of everybody that money will be used as money by everybody else in the economy.

I have no intention to deny the historical possibility that money has actually evolved from some communal mnemonic devices in ancient gift systems.  There are indeed some archaeological evidence suggesting that possibility.[33]  The important thing is, however, to note that at least from the theoretical standpoint there exists a structural discontinuity between the gift system and the monetary system.   It is impossible to fill out this discontinuity by a simple linear causality.  History matters than ever.


*10. On the propagation of the monetary system.*

This paper studied four different decentralized exchange systems – barter system, commodity money system, fiat money system, and gift system – as four different forms of Nash equilibrium in a simple model of decentralized economy.  It showed that while a well-balanced "real" structure of the economy (such as the double coincidence of wants) is necessary to support the barter system as an equilibrium, no such condition (except what we called the "connectedness") is necessary for the commodity money system or the fiat money system or the gift system to do so.   It also showed that

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[33] We can interpret "primitive money" we discussed in footnote 10 as such communal mnemonic devices.

while infinite memory on the part of all the participants of the economy is essential for the gift system to sustain itself as an equilibrium, no memory, not even finite-length memory, is required for the barter system or the commodity money system or the fiat money system to do so. The monetary system, whether it uses commodity money or fiat money, thus occupies a special position among many an exchange system a decentralized economy can have.

Money is money simply because it is used as money by everybody in the economy. Whether it is made of a useful commodity or of a useless token, money is a pure "social entity" whose existence owes nothing to the "technology and preferences" of the economy nor to the "infinite memory" of its members. And it is for this transcendence from "real" as well as "informational" foundations that money can mediate exchanges between individuals whose abilities and needs fail to supplement each other and whose past actions are unknown to each other. But it is precisely for this transcendence from "reality" as well as "informational" foundations that money has fundamental difficulty in evolving naturally from a historically given "real" structure of the economy or from within a small society with tightly-knit interpersonal relationships. The paper indeed argued that the origin of money is theoretically undecidable.

And yet, we human beings are living in a full-fledged monetary economy. No matter how "miraculous" it might be from a purely theoretical standpoint, money did actually emerge on this globe in the distant past, and has since propagated itself all over the globe. It is difficult now to find a society which organizes its economy solely by barter exchanges or by gift exchanges.

In order to conclude this already long paper on the evolution of money, it is therefore necessary to also talk about the fate of monetary system *after* its appearance on this globe. But the story of the propagation of monetary system is qualitatively different from that of the emergence of monetary system. While the latter is about the internal development of a single economy, the former is about

46

competition, communication and interactions among multiple economies.  The former is at least as complex a story as the latter is, and we can only give a brief account of it in the rest of the paper.  In fact, we have to tell two sub-stories – one to economists and the other to anthropologists.

First, imagine a world populated by a large number of economies whose members know only barter exchange,[34] and suppose that one of them has come to be using money. (We of course refrain from delving into the cause of such mutation.)  For simplicity, we suppose that money circulating in this mutant economy is fiat money.   The question we now pose is: would there be any reason for the use of money to propagate itself, once it has come into being at some tiny corner of a barter world?  The answer more or less depends on whether the money-using economy performs better than the rest of the world.  Indeed, there are at least three mechanisms that can work for the propagation of a mutant system – *selection*, *imitation*, and *migration*.  The successful economy has a greater chance of survival and a greater momentum for growth than the less successful ones; the system used in the successful economy is often imitated by the leaders of the less successful economies; and the successful economy tends to attract a large number of immigrants from the less successful neighbors.  Of course, what we compare here is not individuals but economies, so the notion of "success" is a little murky.  But, we can at least say that if the economy with money performs better than the economy without in the sense of Pareto dominance, the use of money is likely to spread out.

Unfortunately, even if the monetary system is autarky-breaking in any connected economy, it is not necessarily Pareto superior to the barter system.  For instance, in our examples of the four-good economy depicted in Fig.1, the monetary system fails to Pareto-dominate the barter system both in the

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[34] Note that, as was discussed in section 5, while some of these economies can sustain barter system as an equilibrium, others cannot do so and may end up with partial barter or complete autarky.

fully-connected economy and in the less-than-fully-but-more-than-minimally-connected economy.[35]

Nonetheless, there is a reason to believe that these cases cannot be regarded as general. Indeed, in the case of minimally-connected economy the monetary system always Pareto-dominates the barter system (because the latter inevitably ends up with complete autarky.) Moreover, it is not difficult to show that even in the cases of fully-connected and less-than-fully-but-more-than-minimally connected economy, if the number of goods $n$ becomes sufficiently large, the monetary system would eventually Pareto-dominate the barter system.[36] All in all, the tendency of the monetary system to propagate itself, once it has come into being in a world originally populated by barter economies, is by no means a necessity but is at least a probability.

Next, imagine another world which is populated by economies whose members exchange their products solely by the system of gift-giving, and suppose that one of them has suddenly come to be using fiat money. We then pose the same question as above: would there be any reason for the use of money to propagate itself, once it has come into being at some tiny corner of this gift-giving world? At first sight, the answer to this question appears to be a simple "no." It is because we know from *Proposition 8* that both fiat money system and gift system perform just identically with respect to the allocation of real goods! Nevertheless, the case is not entirely hopeless, because we also know from *Propositions 11* and *12* that fiat money system and gift system are totally unlike at least with respect to their requirement on the information structure. We need here a slightly more subtle argument than the one given in the case of barter world.

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[35] For instance, in the case of the four-good fully-connected economy it is not hard to show (in fact, we only have to stare at Figs. 4 and 5) that the total search and transaction time in a barter equilibrium can be calculated as $b+12$, whereas the total search and transaction time in a steady-state fiat money equilibrium can be calculated $2b+16$.

[36] For instance, in the case of the $n$-good fully-connected economy the ability-need frequency $e_{ij}$ of the representative individual is given by $1/n(n-1)$, and we can calculate $q_{ij}$ in a barter equilibrium as $1/n(n-1)$ and $q_{i0} = q_{0i}$ in a steady-state fiat money equilibrium as $1/2n$. (In the latter calculation we have to use (20) and (21).) Hence, the total search and

To begin with, the fact that a gift system requires every member of the economy to know every other member's past actions imposes a limit on the size of the economy adopting it. Even if a gift economy performs well, it cannot expand indefinitely and may soon reach a plateau. In stark contrast, a monetary economy knows no such limit and can expand forever. Hence, there still remains a room for the selection mechanism to work here, and the monetary economy may in the long-run overwhelm the gift economy. That is not all. The differential informational structure between monetary system and gift system also has an important implication for the migration mechanism as well. In order to become a member of a gift economy not only do you have to know the past actions of every other member but your past actions also have to be known to every other member. In stark contrast again, all you have to do to join a monetary economy is to accept the money used there as your money and sell your product in exchange. Your personal identity does not matter there. While the migration from a monetary economy to a gift economy provokes a lot of resistance from the members of the latter, the migration from a gift economy to a monetary economy encounters no such resistance. Hence, the monetary economy is likely to grow relatively larger than the neighboring gift economies by constantly absorbing some of their adventurous or disgruntled members. Furthermore, once the monetary economy has outgrown the neighboring gift economies either by the selection mechanism or by the migration mechanism, all sorts of scale and scope economies start to strengthen these mechanisms and may soon invoke the imitation mechanism as well. Again, the tendency of the monetary system to propagate itself, once it has come into being in the world originally populated by gift economies, is by no means a necessity but is at least a probability.

As the length and tortuosity of this paper have indicated, the evolution of money is certainly a long, tortuous process.

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transaction time in the former is equal to $b+n(n-1)$ and in the latter is equal to $2b+4n$. Hence, if $n(n-5) > b$, the fiat money

equilibrium Pareto-dominates the barter equilibrium.

*Appendix: The optimal exchange strategy for infinitely-lived consumers.*

The purpose of this *Appendix* is to show that even if the basic decision units in our decentralized economy are not finitely-lived consumers but infinitely-lived consumers (or infinite-horizon family dynasties), there is no need to change any of the results reported in this paper.

Consider an infinitely-lived *i*-producing *j*-consumer. Let us assume that he receives a positive utility $u$ $(> 0)$ from consuming a unit of good $j$ but no utility from any other good, including his own product. We also assume that he discounts future utilities by a positive time discount rate $r$ $(> 0)$. For simplicity, we assume that neither production nor exchange takes any time, though they can be easily incorporated into the model. We further assume that his expectations about future parameters are all stationary, so that we can work only on the steady-state strategies. One trivial consequence of these assumptions is that the life-time expected utility of staying in autarky is zero.

Let $V_{i\emptyset j}$ denote the expected discounted life-time utility of an *i*-producing *j*-consumer with a product $i$ in hand, when he has committed to barter. Then, the method of dynamic programming allows us to calculate its value as follows. Consider a very small time interval $\Delta$ $(>0)$. With a probability $q_{ji}\Delta$, he meets a trading partner, obtains a unit of good $j$ in exchange of good $i$, enjoys a utility of $u$ by consumption, produces a unit of good $i$, and then starts a search activity again with the same utility prospect $V_{i\emptyset j}$ as before. His total discounted utility in this case is therefore equal to $(u+V_{i\emptyset j})/(1+r\Delta)$. On the other hand, with a probability $1-q_{ji}\Delta$, he fails to meet a trading partner. His total discounted utility is $V_{i\emptyset j}/(1+r\Delta)$ in this case. Hence, $V_{i\emptyset j}$ can be expressed self-referentially as:
$V_{i\emptyset j} = \{q_{ji}\Delta(u+V_{i\emptyset j}) + (1-q_{ji}\Delta)V_{i\emptyset j}\}/(1+r\Delta)$. Solving this, we obtain $V_{i\emptyset j} = uq_{ji}/r$: For later convenience, we rewrite this as: $V_{i\emptyset j} = u/\{(1+r/q_{ji})-1\}$.

Next, let $V_{ikj}$ denote the expected discounted life-time utility of our *i*-producing *j*-consumer with his product in hand, when he is using good $k$ $(\neq i, \ j)$ as the sole medium of exchange. Let us also denote by $V_{ikj}'$ his expected life-time utility when he holds money in hand. Then, the method of dynamic programming again allows us to relate these two life-time utilities during $\Delta$ $(>0)$ as: $V_{ikj} = \{q_{ki}\Delta V_{ikj}' + (1-q_{ki}\Delta)V_{ikj}\}/(1+r\Delta)$, and $V_{ikj}' = \{q_{jk}\Delta(u+V_{ikj}) + (1-q_{jk}\Delta)V_{ikj}'\}/(1+r\Delta)$. Solving them, we obtain for every $k$ $(\neq i, j)$: $V_{ikj} = u/\{(1+r/q_{ki})(1+r/q_{jk})-1\}$.

In general, let $V_{ikl..hj}$ denote the expected discounted utility of our $i$-producing $j$-consumer with his product in hand, when he has committed to an indirect exchange which uses goods $k, l, ..., g, h$ as media of exchange. We can calculate it as: $V_{ikl...ghj} = u/\{(1+r/q_{ki})(1+r/q_{lk})\cdots(1+r/q_{hg})(1+r/q_{jh})-1\}$ .

Note here that all the above expressions have the form of $u/(D-1)$. Since the maximization of $u/(D-1)$ is equivalent to the minimization of $D$ or $log(D)$, and since the non-autarky condition $u/(D-1) > 0$ is equivalent to $log(D) < \infty$, we can formulate the optimal search program for our producer-consumer in the form of:

*Lemma A*: The optimal search program for an infinite-living $i$-producing $j$-consumer is to choose a set of indices, $k, l,..., g, h,$ which minimizes the following summation:

$log(1+r/q_{ki})+log(1+r/q_{lk})+\cdots+log(1+r/q_{hg})+log(1+r/q_{jh})$.

If the minimum value is infinite, he rather stays in autarky. If the minimizing set of indices is null (ø), he seeks barter exchange. Otherwise, he seeks an indirect exchange which uses the minimizing set of goods, $k, l,..., g, h$, as his media of exchange. □

Now, the term $log(1+r/q_{kl})$ in the above formula can be regarded as an index of search cost in $(l, k)$ trading zone. (If $r$ is very small, it indeed can be approximated as $r/q_{kl}$.) Then, our infinitely-lived producer-consumer can determine his optimal exchange program simply by adding this index of search cost in each trading zone and minimizing their total sum. It is as if he had a finite life-time and no time discounting, as in the model of the main text.

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